Extreme events on graphs

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April 2026

Journée scientifique Geolearning

1. Context

The cost of simultaneous river floods

2021

July floods

Germany, Belgium, Netherlands, Luxembourg

€46B

Economic losses

Insured & uninsured damages across affected countries

Understanding spatial dependence of extreme floods is key for risk assessment

Which European rivers are more likely to flood *simultaneously*?

Understanding the spatial dependence of extreme river flows

PhD objective: Build a model to generate extreme river flows

New graphical model
Nodes : weather and river flow variables

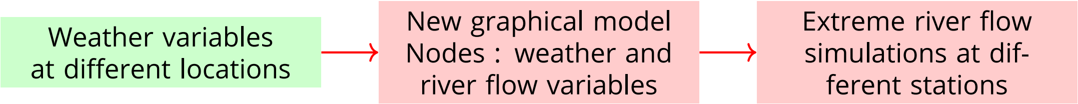
PhD objective: Build a model to generate extreme river flows

Weather variables
at different locations

→

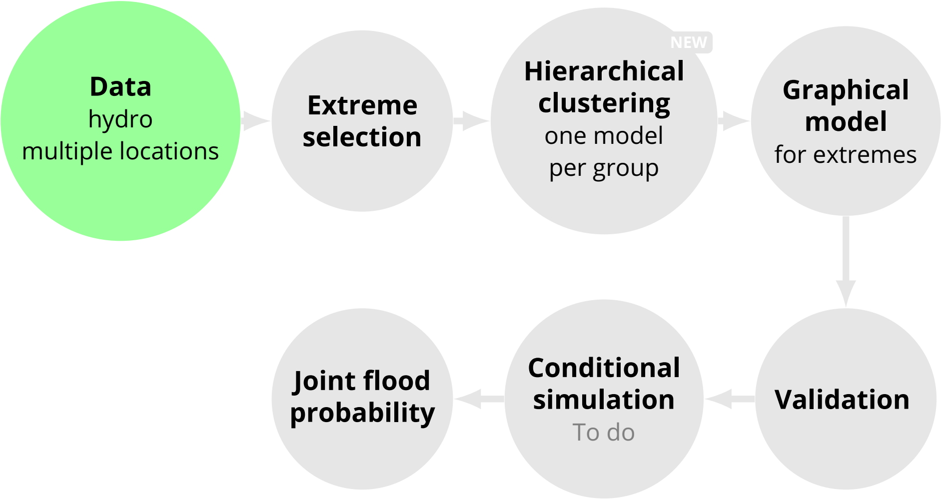
New graphical model
Nodes : weather and
river flow variables

PhD objective: Build a model to generate extreme river flows

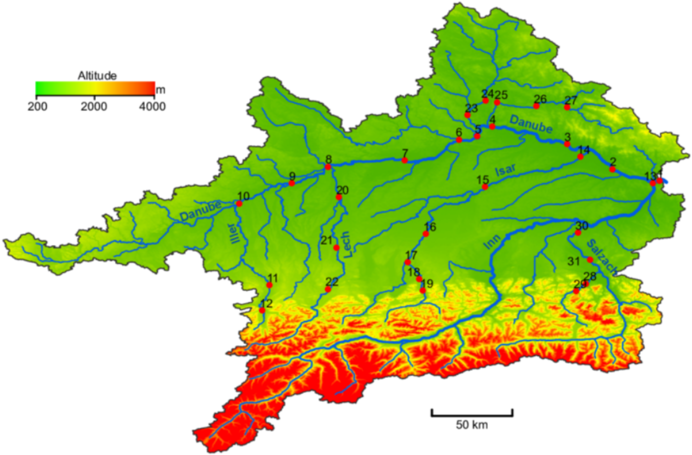


2. Application

Workflow to model multivariate extremes



Danube River network in Germany Source: Asadi et al. (2015)



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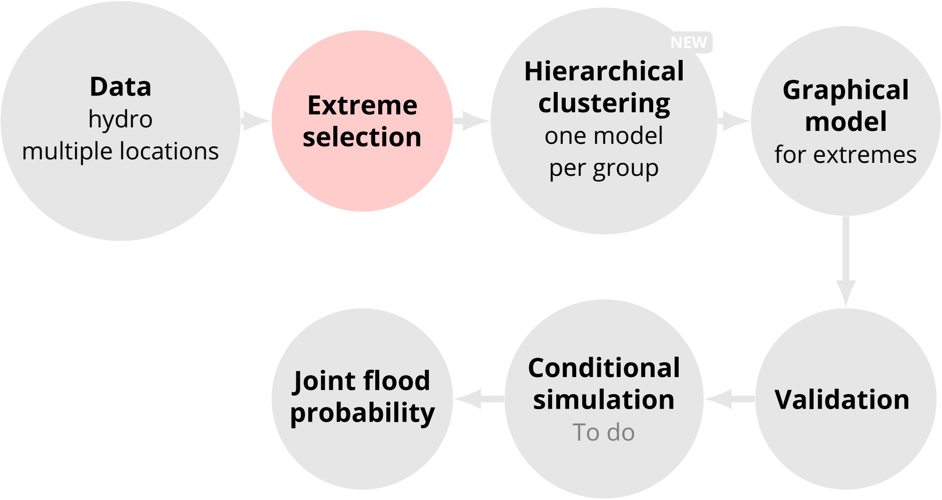

Extreme selection

Data

Riverflow
daily data
1900–2014
31 stations
$(X_1, \ldots, X_{31})$

01

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Marginal transformation

$X_i^* = \frac{1}{1-F_i(X_i)} \sim \text{Pareto}(1)$

02

Why this matters

- Each $X_i \sim F_i$ has its **own marginal distribution**
- $X_i^* = \frac{1}{1-F_i(X_i)} \sim \text{Pareto}(1)$
- $\Rightarrow$ All X_i^* share a **common reference distribution** $\Rightarrow$ variables are **comparable**

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Extreme selection

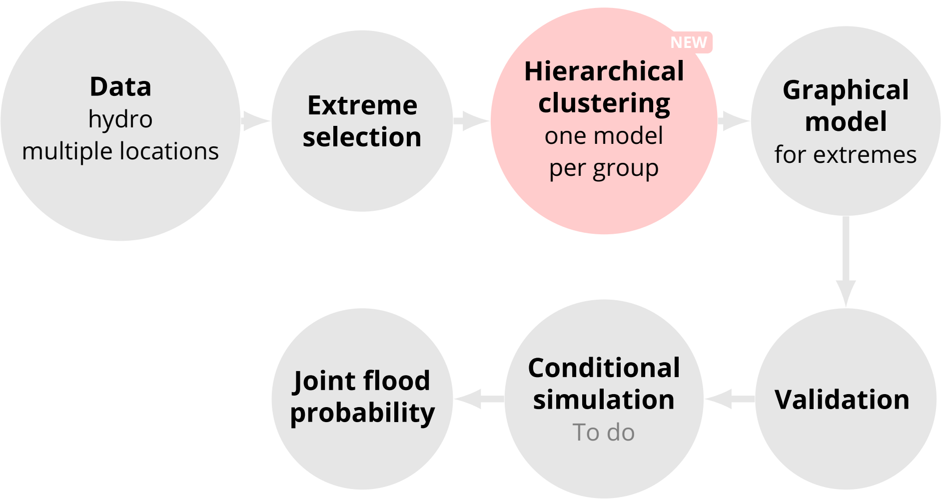
$\exists i : X_i^* > u$

$\mathbf{Y} = \frac{\mathbf{X}^*}{u}$

03

$\mathbf{Y}^1, \mathbf{Y}^2, \ldots, \mathbf{Y}^n$ are assumed i.i.d. and follow a **MGPD** (Multivariate Generalized Pareto Distribution)

Workflow to model multivariate extremes



Hierarchical clustering - grouping dependent variables

Distance between two variables

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Distance between two variables

- $\hat{\chi}_{ij}$ = empirical tail dependence

$$\chi_{ij} = \mathbb{P}(X_j^* > u \mid X_i^* > u) \in [0, 1]$$

$$= \mathbb{P}(Y_j > 1 \mid Y_i > 1) \in [0, 1]$$

χ_{ij} measures the probability that variable *i* is extreme **given that** variable *j* is extreme.

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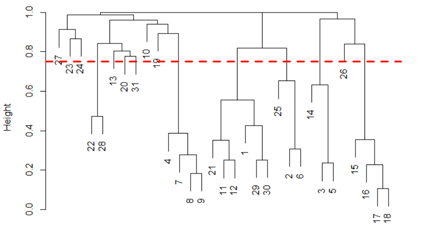
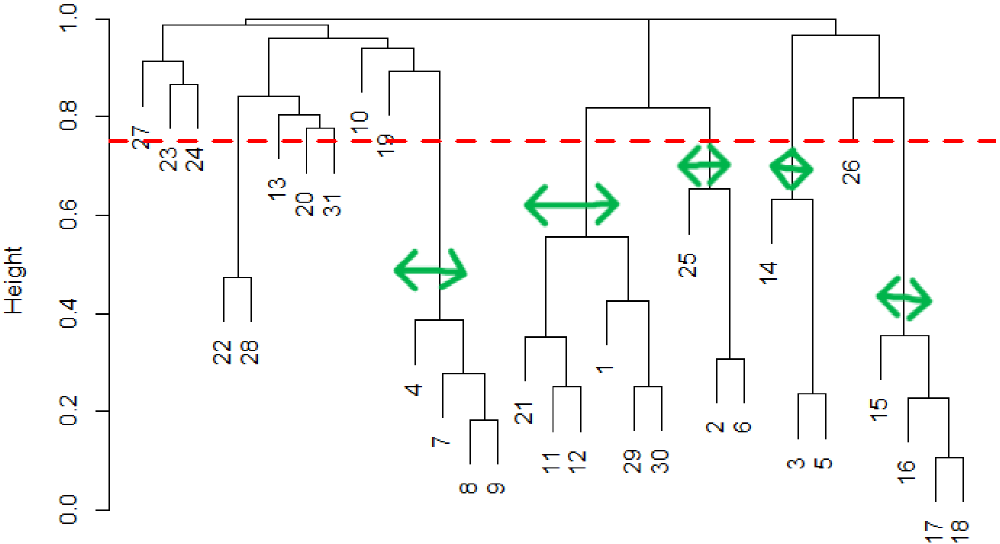
Result

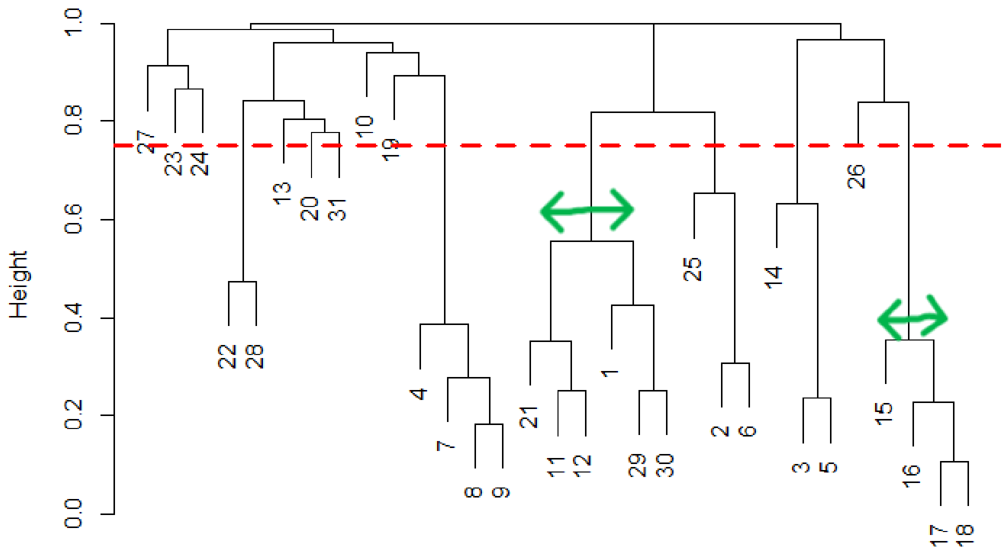


- Each leaf represents one variable
- Cut tree at threshold $\Rightarrow$ clusters of dependent variables
- Variables that merge at low height $\Rightarrow$ strong tail dependence

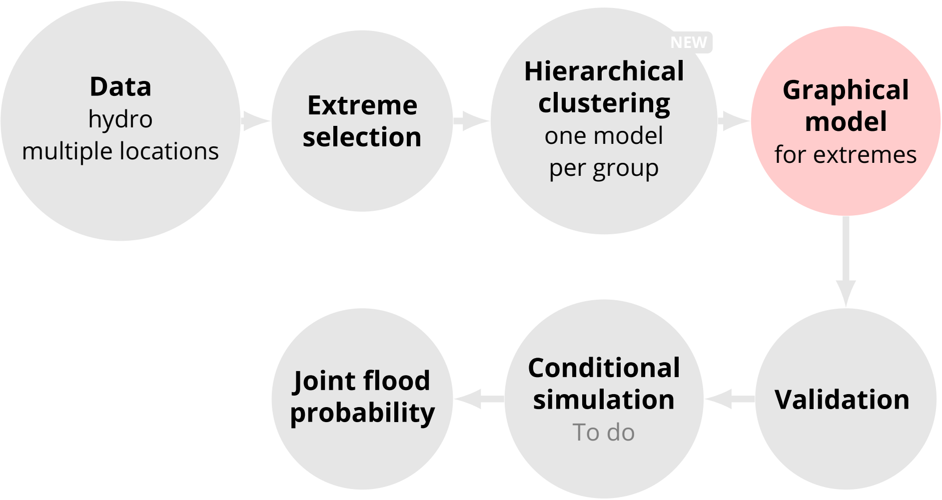
Hierarchical clustering - grouping dependent variables



Example of application : selection of 2 clusters



Workflow to model multivariate extremes



Graphical model for multivariate extreme events: Hüsler–Reiss

- **Hüsler–Reiss:** Gaussian analogue for multivariate extremes $\mathbf{Y} = (Y_1, \dots, Y_d)$

$$\lambda(\boldsymbol{y}) \propto \prod_{i=1}^{d} y_i^{-(1+1/d)} \exp\left(-\frac{1}{2}(\log \boldsymbol{y} - \boldsymbol{\mu}_\Theta)^T \boldsymbol{\Theta} (\log \boldsymbol{y} - \boldsymbol{\mu}_\Theta)\right)$$

$\rightarrow$ Characterised by precision matrix $\boldsymbol{\Theta} = (\theta_{ij})_{1 \leq i,j \leq d}$

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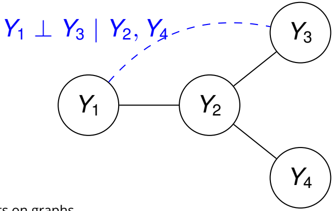
$\rightarrow$ Characterised by precision matrix $\boldsymbol{\Theta} = (\theta_{ij})_{1 \le i,j \le d}$

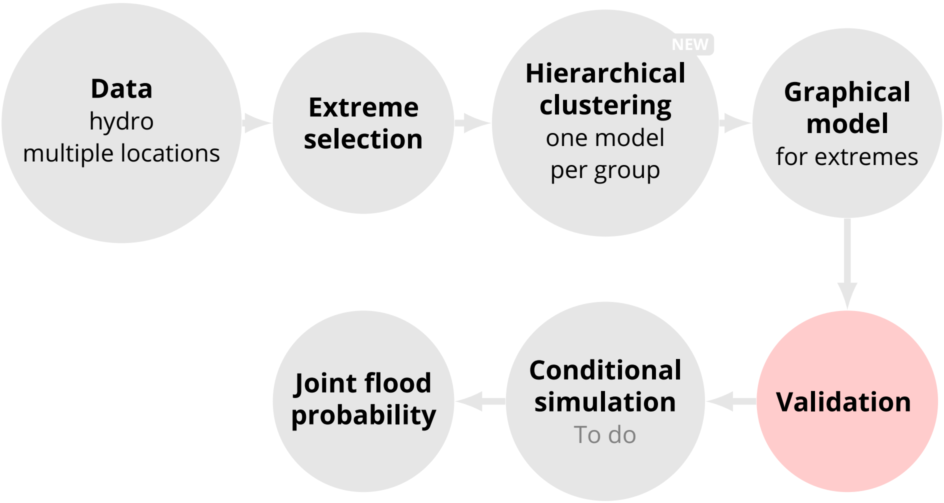
- **Graphical model:** Θ encodes dependencies via a graph $G = (V, E)$

$$\theta_{ij} = 0 \iff \text{no edge } (i,j) \iff Y_i \perp Y_j \mid \mathbf{Y}_{-ij}$$



$$\boldsymbol{\Theta} = \begin{pmatrix} \theta_{11} & \theta_{12} & 0 & 0 \\ \theta_{12} & \theta_{22} & \theta_{23} & \theta_{24} \\ 0 & \theta_{23} & \theta_{33} & 0 \\ 0 & \theta_{24} & 0 & \theta_{44} \end{pmatrix}$$

Workflow to model multivariate extremes



First validation criteria

Tail dependence - χ measure

$$\chi_{ij} = \mathbb{P}(X_j^* > u \mid X_i^* > u) \in [0, 1]$$

$$= \mathbb{P}(Y_j > 1 \mid Y_i > 1) \in [0, 1]$$

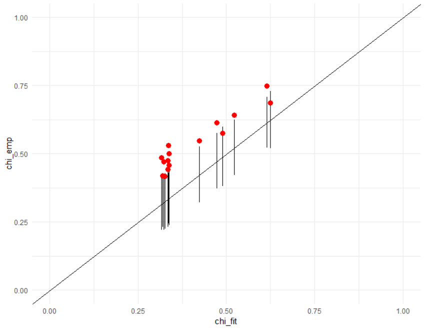
χ_{ij} measures the probability that variable i is extreme **given that** variable j is extreme.

Empirical $\hat{\chi}ij$ vs. χij computed from the fitted model

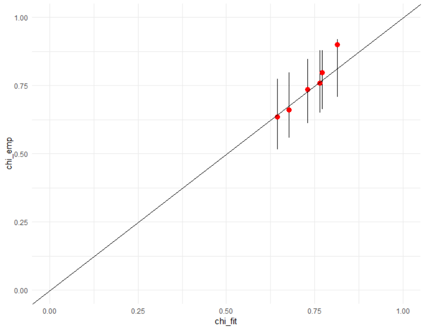
First validation criteria

Idea: if the model is correct, empirical $\hat{\chi}_{ij}$ should match χ_{ij} computed from the fitted model. Points close to diagonal $=$ good fit

χ measure for stations $(1-11-12-21-29-30)$- ✓ Good



χ measure for stations $(15-16-17-18)$ - ✓ Good



Second validation criteria

Conditional normality (Hüsler–Reiss property)

Model property:
Under the Hüsler–Reiss model, for any conditioning variable m,

$$\mathbf{Z} = (\log Y_i - \log Y_m)_{i \neq m} \,\Big|\, Y_m > 1$$

$$\sim \mathcal{N}\left(-\tfrac{1}{2}\,\mathrm{diag}(\Sigma^{(m)}),\ \Sigma^{(m)}\right)$$

$\Rightarrow$ Define **standardized residuals**:

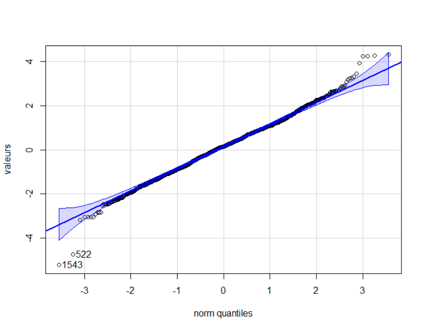
$$\mathbf{R} = (\Sigma^{(m)})^{-1/2}\left[Z + \tfrac{1}{2}\,\mathrm{diag}(\Sigma^{(m)})\right]$$

$$\sim \mathcal{N}(0, I)$$

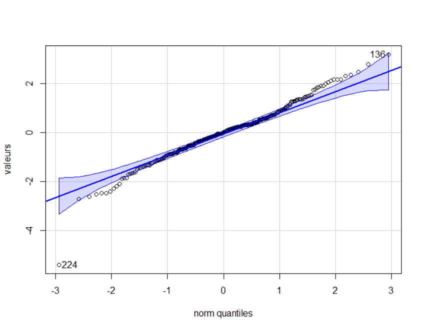
Second validation criteria

Idea: if the model is correct, R should be Gaussian. Points close to diagonal $=$ good fit

Q-Q plots for stations $(1 - 11 - 12 - 21 - 29 - 30)$ **-** ✓

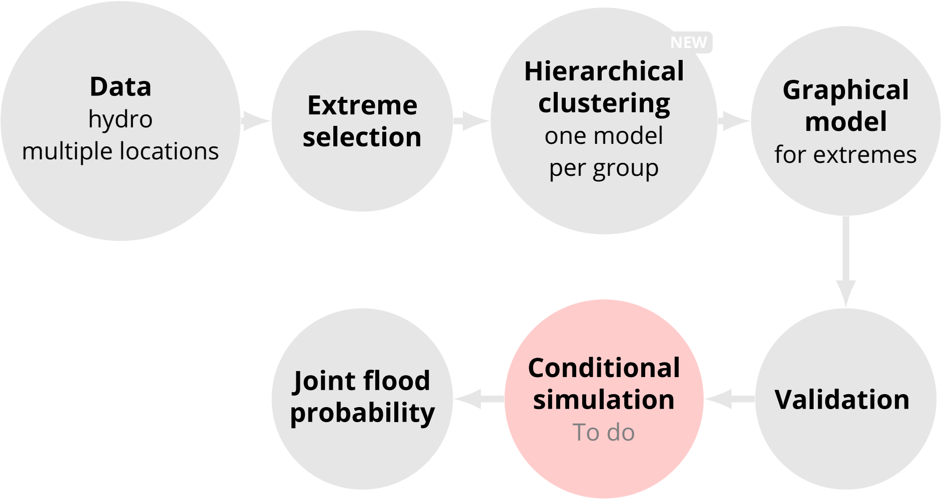


Q-Q plots for stations $(15 - 16 - 17 - 18)$ **-** ✓



3. Perspectives

Workflow to model multivariate extremes



Perspectives

Conditional simulation: currently validated on training data - next step is to evaluate on test data for true out-of-sample assessment

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Scale up: include climate variables and river flows together - extend to larger European river networks

Thank You

Rita Maatouk
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April 2026

References

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4. Annexe

Methodological pipeline

Data

Rainfall/Soil moisture daily data 1970–2024
6 watersheds
$(X_1, \ldots, X_6)$

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Declustering

Extract independent extreme events **Y** based on a threshold u

03

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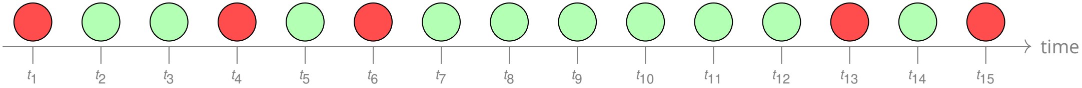
Goal of declustering

Retain **independent** extreme events - consecutive extremes may be part of the **same episode**

Declustering: extracting independent extreme events

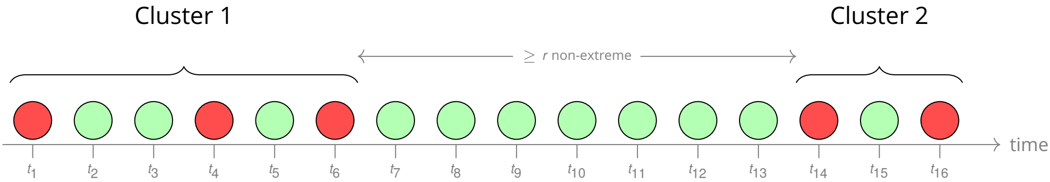
- Each observation = one row $(X_1^*, \ldots, X_6^*)$ in the dataset
- Threshold u typically chosen as a high quantile



● **Extreme:** $\exists\, i : X_i^* > u$; ● **Non-extreme:** $\forall\, i : X_i^* \leq u$

Declustering: extracting independent extreme events

- We fix a constant r (here $r = 5$)
- Two extremes $\rightarrow$ **same cluster** if $< r$ non-extreme obs. between them
- Two clusters $\rightarrow$ **distinct** if $\geq r$ non-extreme obs. between them



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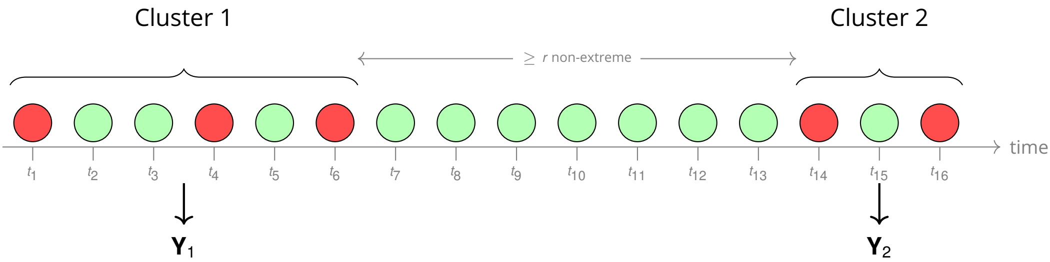

- **Extreme:** $\exists\, i : X_i^* > u$; - **Non-extreme:** $\forall\, i : X_i^* \leq u$
- $\mathbf{Y}^k = \left(\frac{\max_{t \in C_k} X_{t1}^*}{u}, \ldots, \frac{\max_{t \in C_k} X_{t6}^*}{u} \right) = (\mathbf{Y}_1^k, \ldots \mathbf{Y}_6^k) \in \mathbb{R}^6$
- $\mathbf{Y}^1, \mathbf{Y}^2, \ldots, \mathbf{Y}^n$ are assumed i.i.d. and follow a **MGPD** (Multivariate Generalized Pareto Distribution)

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Extract independent extreme events **Y** based on a threshold u

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Graphical model

Fit Hüsler–Reiss on extremes **Y**

Graphical model for multivariate extreme events

A way to represent multiple variables and their relationships using a graph

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A way to represent multiple variables and their relationships using a graph

Summary	Gaussian model	**Hüsler–Reiss model**
Density	$\propto \exp(-\frac{1}{2}\|\mathbf{x}-\mu\|_Q^2)$	$\propto \exp(-\frac{1}{2}\|\log(\mathbf{y})-\mu_Q\|_Q^2)$

where $\mu_Q = P\left(-\frac{\Gamma}{2}\right)\mathbf{1}$ with $P = I - \frac{1}{d}\mathbf{1}\mathbf{1}^\top$

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Precision matrix Q	Σ^{-1}	$\left(P(-\frac{1}{2}\Gamma)P\right)^{+}$

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Representative observation per cluster

- A cluster C_k contains several observations:

$$\mathbf{X}^*_{t_1},\ \mathbf{X}^*_{t_2},\ \ldots,\ \mathbf{X}^*_{t_m}$$

with

$$\mathbf{X}^*_t = (X^*_{t1}, \ldots, X^*_{td})$$

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- We summarize the cluster by taking the maximum reached by each variable

$$\begin{aligned} \mathbf{X}^*_{t_1} &= (X^*_{t_1 1}, \ldots, X^*_{t_1 d}) \\ \mathbf{X}^*_{t_2} &= (X^*_{t_2 1}, \ldots, X^*_{t_2 d}) \\ &\vdots \\ \mathbf{X}^*_{t_m} &= (X^*_{t_m 1}, \ldots, X^*_{t_m d}) \end{aligned} \longrightarrow \mathbf{Y}_k = \frac{1}{u}\left(\max_{t \in C_k} X^*_{t1}, \ldots, \max_{t \in C_k} X^*_{td}\right)$$

As $u \to \infty$, the vectors $\mathbf{Y}_k$ follow a Multivariate Generalized Pareto Distribution (MGPD).

Graphical model for extreme events : Hüsler–Reiss HR(Γ)

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Parameter estimation strategies

Fixed graph

Graph is imposed
Estimate model parameters

Parameter estimation strategies

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Data-driven

Estimate the graph
Estimate model parameters

Parameter estimation strategies

Fixed graph

Graph is imposed
Estimate model parameters

Data-driven

Estimate the graph
Estimate model parameters

Parameter estimation strategies

Estimation of HR(Γ) model parameters

↓

Maximize surrogate log-likelihood

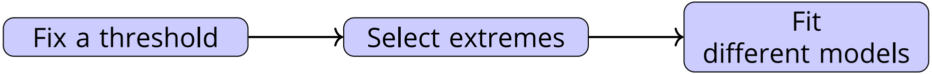
$l(Q) = \log\det(Q) + \frac{1}{2}\mathrm{Tr}(\hat{\Gamma} Q)$

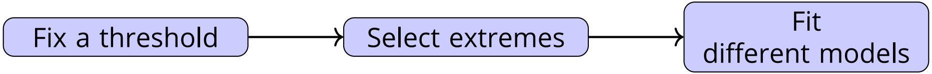
Approach with a fixed graph

1. Zero pattern of Q is imposed
2. Maximization over non-zero entries

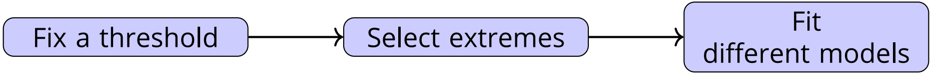
Data-driven approach

1. Zero pattern of Q is estimated
2. Maximization over non-zero entries


Fix a threshold
Select extremes
Fit
different models



$\rightarrow$ Iterate over different model specifications and threshold levels



$\rightarrow$ Iterate over different model specifications and threshold levels
$\rightarrow$ Select and validate an appropriate model

Validation Criteria

1. The number of selected extreme events is reasonable (bias-variance tradeoff)

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2. Check the conditional multivariate normality suggested by the model:

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   Transformed remaining variables $\big|$ one variable is extreme $\approx$ Gaussian distribution

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Validation Criteria

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3. Another way to assess the reliability of the model is to compare the dependence observed in the data with that predicted by the model using the χ measure
4. The χ measure indicates whether two variables tend to experience extreme events simultaneously
5. For each pair of variables, we compare:
   - the empirical χ estimated from the data
   - the fitted χ predicted by the model

Results

- **Edge thickness** = magnitude of partial correlation $\left|\frac{\theta_{ij}}{\sqrt{\theta_{ii}\theta_{jj}}}\right|$
  thicker = stronger dependence
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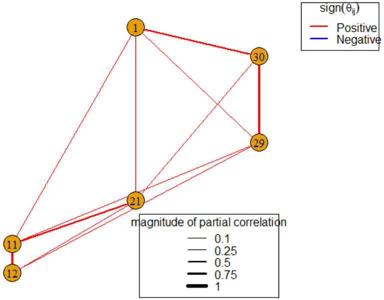
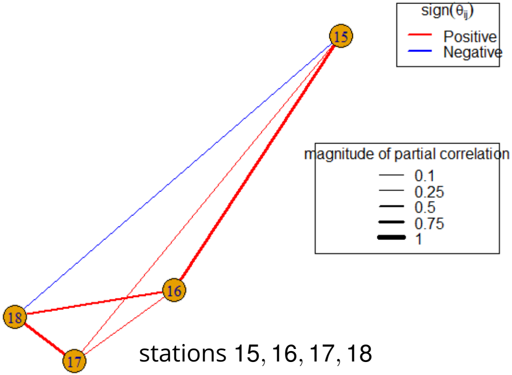


stations 1, 11, 12, 21, 29, 30



stations 15, 16, 17, 18