Analyzing temporal dependence between extreme events using point processes

joint work with D. Allard, E. Gabriel and T. Opitz

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Natural hazards in 2022 in France

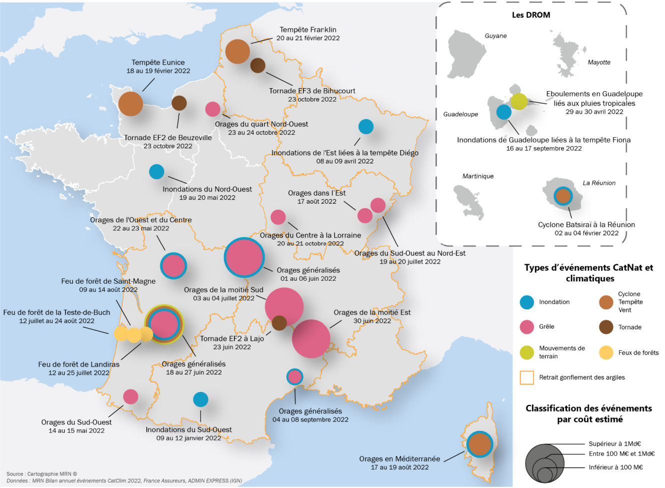

Figure: Source: Mission Risques Naturels

Motivation

- **Context:** Transforming "big data" from climate observation and modeling into actionable knowledge on Compound Extreme Events
- **Challenges:** Use conceptual framework and methods from point processes and extreme value theory
- **Objectives:** Understand and model the spatial and temporal (co-)occurrence of extreme events focusing on :
  - event representation,
  - subdivision of geographic space,
  - risk analysis

Marked Point Processes for Extreme Events

- **Event-based framework:**
  - Extreme events represented as discrete occurrences in time, space, or space-time.
  - Strong data reduction by retaining only key event features.
- **Marked point processes:**
  - A **point**: event location.
  - A **mark**: associated variables (magnitude, duration, intensity).
- **Extreme-value link:** High-threshold exceedances converge to specific point processes (Pickands III 1971; Hsing, Hüsler, and Leadbetter 1988.)
- **Key characteristics:**
  - First-order: intensity (mean arrival rate).
  - Second-order: clustering or repulsion (e.g. Ripley's K).

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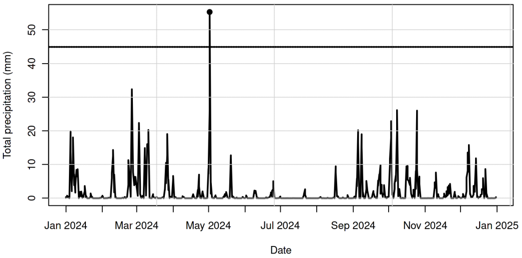
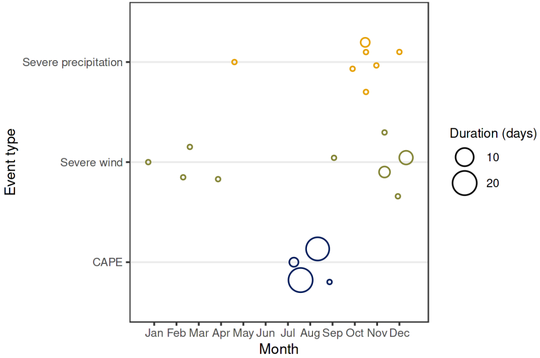
Overview of the Study

- We apply a unified **temporal point process framework** to characterise short-term dependencies between extreme meteorological events across Europe.
- Two complementary perspectives illustrate different facets of **compound events**:
  - **Preconditioned extremes**: wildfire-related weather conditions and short-term risk windows (FWI and meteorological drivers).
  - **Multivariate and spatial compounding**: temporal clustering between extreme precipitation, wind, and CAPE within and across European regions.
- All analyses rely on **ERA5 reanalysis data** (from ECMWF), aggregated to daily scale and consistently applied across regions.
- We combine second-order statistics (e.g. cross-K functions) with simulation-based tests to assess departures from temporal independence.

Construction of Extreme Events





- High local quantile exceedances at each grid point ($q_{0.95}$ or $q_{0.999}$).
- Nearby exceedances in space and time are grouped.
- Each connected cluster forms one **event** (one point).

Point Process : Intensity & Temporal K-Function

- **First-order: Intensity**
  - Non-stationary:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{\mathbb{E}[N([t, t + \Delta t])]}{\Delta t}$$

  - Seasonal (climate):

$$\hat{\lambda}(u) = \bar{n}\,\hat{f}(u)$$

    periodic intensity (DOY, circular kernel)

- **Second-order: Temporal K-function**
  - Measures number of events within lag r:

$$K(r) = \frac{1}{\lambda}\mathbb{E}[N([t_0 - r, t_0 + r]) \mid t_0]$$

  - Inhomogeneous version: accounts for $\lambda(t) \to$ removes seasonal effects

Cross-K Functions (Bivariate Temporal):

- **Cross-K (dependence between N_x, N_y):**
  - Expected number of y-events within lag r of an x-event
  - **Non-homogeneous estimator**, from (Cebrián, Abaurrea, and Asín 2020) :

$$\hat{K}_{xy}(r) = \frac{1}{T_{\text{tot}}} \sum_{t_i \in N_x} \sum_{s_j \in N_y} \frac{\mathbf{1}(|t_i - s_j| \leq r)}{\hat{\lambda}_x(t_i)\hat{\lambda}_y(s_j)}$$

  - DOY-based intensities (circular kernel) to removes seasonal effects
- **Directional cross-K ($x \to y$)**

$$\hat{K}_{x \to y}(r) = \frac{1}{T_{\text{tot}}} \sum_{t_i \in N_x} \sum_{s_j \in N_y} \frac{\mathbf{1}(0 \leq s_j - t_i \leq r)}{\hat{\lambda}_x(t_i)\hat{\lambda}_y(s_j)}$$

  - Counts only y after x (precursor detection)

Testing Temporal Independence

Null hypothesis:

$$H_0 : N_x \perp\!\!\!\perp N_y$$

Monte Carlo strategy:

- Keep N_x fixed, resample N_y
- Compare observed cross-K to simulations

Null models

- **Translation:**

$$u'_j = (u_j + \Delta_y) \bmod 1$$

  preserves seasonality + clustering, breaks alignment

- **NHPP simulation:**
  - Simulate from $\hat{\lambda}_y(u)$ (thinning)
  - preserves intensity only (no clustering)

Global Envelope Test (GET)

Test statistic :

$$\hat{K}_{xy}(r) - 2r \quad \text{or} \quad \hat{K}_{x\to y}(r) - r = 0 \quad \text{under} \quad H_0$$

Procedure:

- Simulate n_{sim} patterns under H_0
- Build global rank envelope from (Myllymäki and Mrkvička 2024)

Decision:

- Outside envelope $\rightarrow$ reject independence
- Inside envelope $\rightarrow$ not rejected

Interaction between Meteorological Events in Europe

Objective:

- Study temporal interactions between extreme **precipitation, wind, and convection**
- Focus on compound and high-impact events
- Event-based point process framework

Data (ERA5, 1941–2024):

- Total precipitation (daily sum) — $q_{0.999}$ of wet days (> 1 mm)
- 10-m wind speed (daily max) — $q_{0.999}$
- CAPE (daily max) — $q_{0.999}$

Extreme definition:

- Long-term stationary thresholds (1941–2024)
- Target rare, high-impact events
- No trend analysis — focus on temporal dependence

Inter- vs Intra-Regional Dependence

Common framework: translation-based null model (seasonality preserved, alignment broken).
Maps display $-\log_{10}(p)$ to quantify strength of temporal dependence.

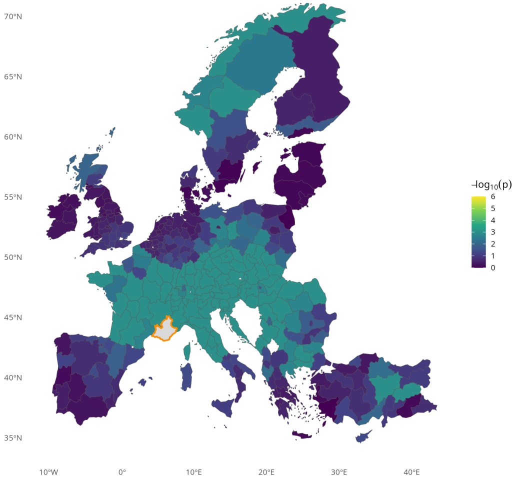
Inter-regional (same variable)

- Compare each NUTS-2 region to a reference
- Same hazard (e.g. wind–wind)
- Test temporal co-occurrence within $r = 3.5$ days
- Two-sided Global Envelope Test

Intra-regional (cross-variable)

- Within one NUTS-2 region
- Different hazards (e.g. wind–precipitation)
- Same temporal radius $r = 3.5$ days
- Two-sided Global Envelope Test

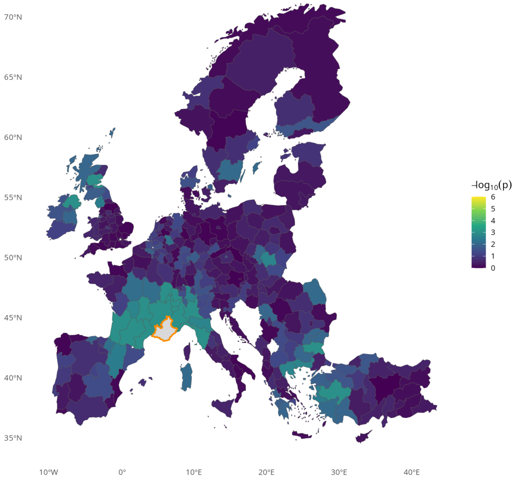
Inter-event dependence test between severe wind extremes and FRL0



Interpretation

- High $-\log_{10}(p)$
- Strong dependence
- Spatial coherence
- Central & Southern Europe

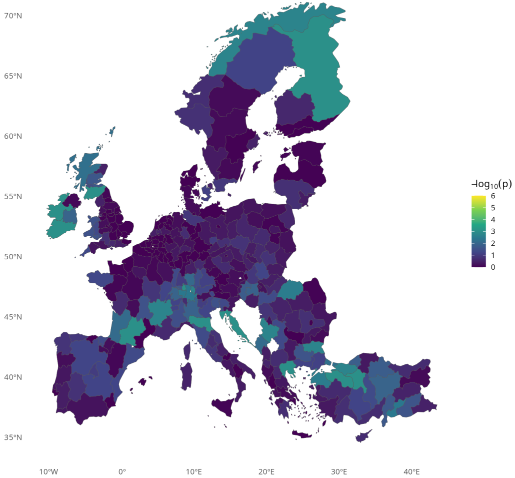
Inter-event dependence test between severe precipitation extremes and FRL0



Interpretation

- Fragmented pattern
- Local dependence
- Neighboring regions
- Weak spatial coherence

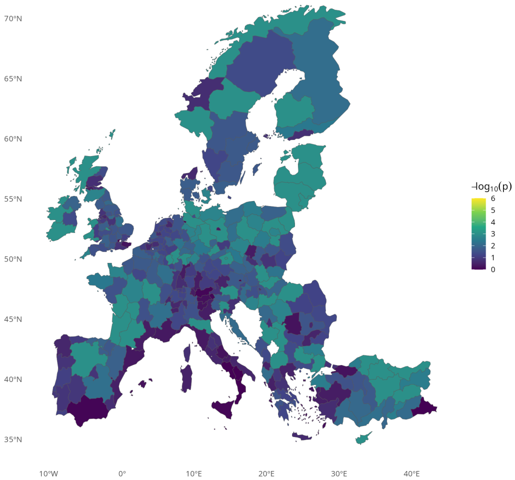
Intra-regional dependence test of CAPE preceding severe wind



Interpretation

- Regional effect
- CAPE as precursor
- Mountain regions
- Localised signal

Intra-regional dependence test of CAPE preceding severe precipitation



Interpretation

- Widespread signal
- Strong dependence
- Europe-wide pattern
- Local weakening

Conclusion and Perspectives

Key findings

- Work submitted, see for more details, Heranval et al. 2026
- Event-based point process framework for extreme interactions
- Cross-K + Global Envelope Tests for temporal dependence
- Severe wind: broad inter-regional temporal coherence
- Precipitation: more localised dependence structure
- Intra-regional wind–precip coupling frequent
- CAPE interactions weaker and region-specific (notably mountainous areas)

Perspectives

- Marked and fully spatio-temporal K-functions
- Integration with climate circulation indices
- Link with impact datasets for compound-risk assessment

Thank You for Your Attention

Analyzing temporal dependence between extreme events using point processes. Antoine Heranval, Denis Allard, Edith Gabriel, Thomas Opitz. 2026

References I

- Cebrián, Ana C, Jesús Abaurrea, and Jesús Asín (2020). "Testing independence between two nonhomogeneous point processes in time". In: *Journal of Statistical Computation and Simulation* 90.16, pp. 2878–2901. DOI: 10.1080/00949655.2020.1792471.
- Heranval, Antoine et al. (2026). "Analyzing Temporal Dependence Between Extreme Events Using Point Processes". HAL preprint, https://hal.science/hal-05519299.
- Hsing, Tailen, Jürg Hüsler, and Malcolm Ross Leadbetter (1988). "On the exceedance point process for a stationary sequence". In: *Probability Theory and Related Fields* 78.1, pp. 97–112. DOI: 10.1007/bf00718038.
- Myllymäki, Mari and Tomáš Mrkvička (2024). "GET: global envelopes in R". In: *Journal of Statistical Software* 111, pp. 1–40.
- Pickands III, James (1971). "The two-dimensional Poisson process and extremal processes". In: *Journal of applied Probability* 8.4, pp. 745–756. DOI: 10.1017/s0021900200114640.