Statistical modeling of spatio-temporal data distributed over surfaces

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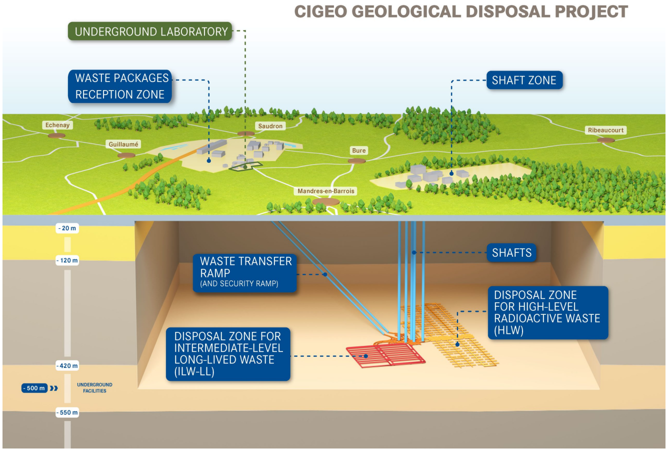
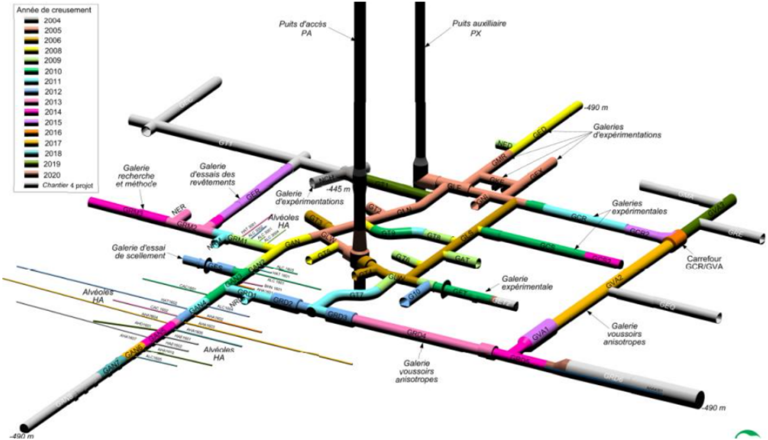
CONTENT

■ CIGEO PROJECT



■ OBJECTIVES

Long-term objective: data monitoring of disposal galleries

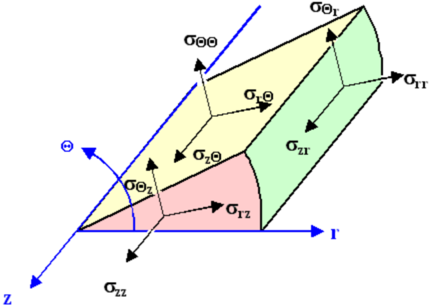


UNDERGROUND LABORATORY: DATA OVERVIEW

Continuous Monitoring since 2017

Daily multi-sensor data acquisition focusing on two primary quantities of interest:

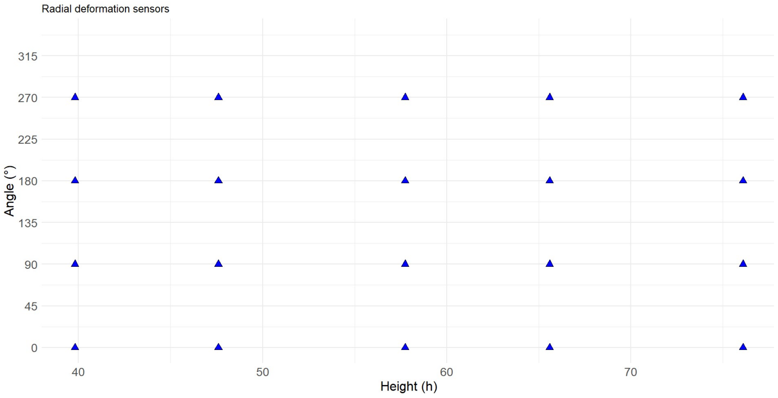
- **Temperature**
- **Deformation** (3-component analysis):
  - Radial ← *Study focus*
  - Orthoradial
  - Longitudinal



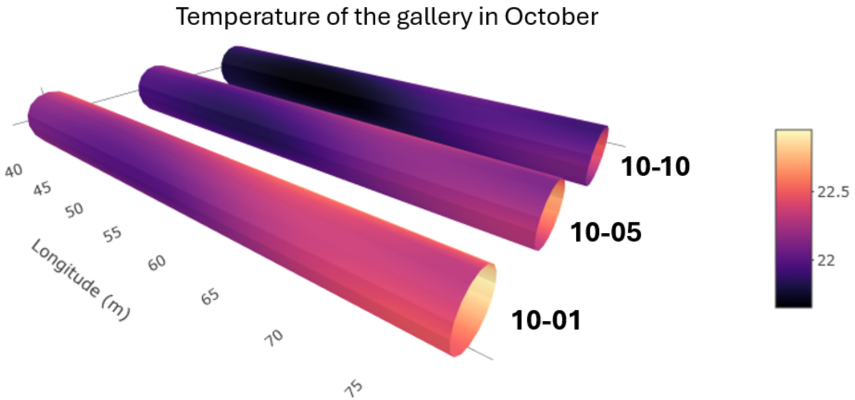
Gallery deformation components

■ SENSORS POSITION

One deformation sensor $\Leftrightarrow$ One temperature sensor



■ SPATIO-TEMPORAL FIELDS

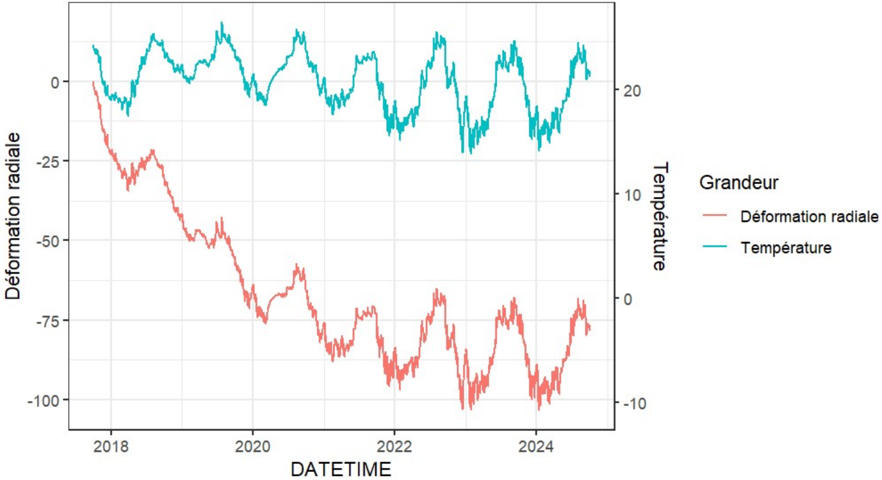


RESEARCH QUESTIONS

- Cylindrical surface with complex geometry
- Spatio-temporal data
- Coupled variables: temperature and deformation are interdependent

CONTENT

■ SIGNALS



0
-25
-50
-75
-100
Déformation radiale
20
10
0
-10
Température
2018
2020
2022
2024
DATETIME
Grandeur
Déformation radiale
Température

SPATIO-TEMPORAL MODEL

Typical spatio-temporal model

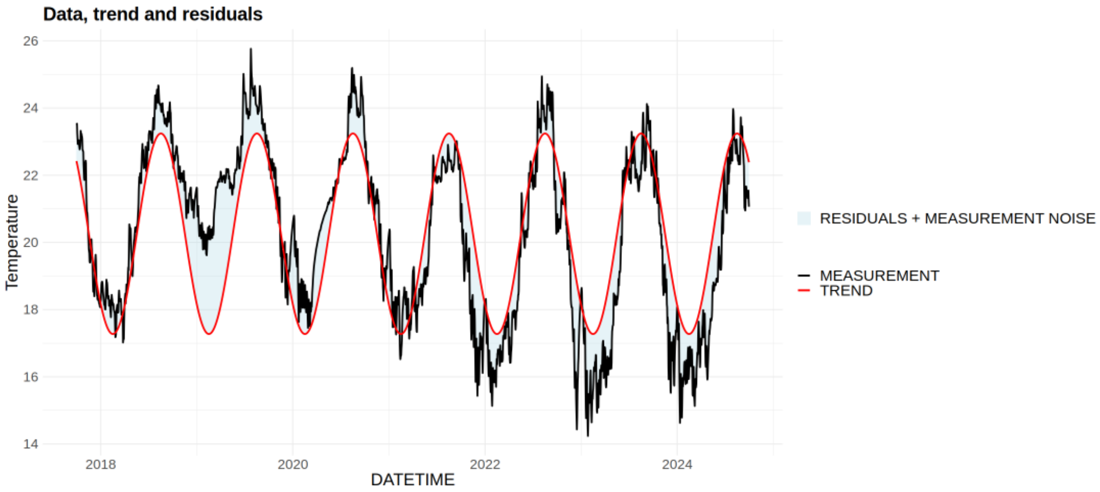
$$\begin{cases} Y_{\text{Temp}}(t, \mathbf{s}) = \mu_{\text{Temp}}(t, \mathbf{s}) + X_{\text{Temp}}(t, \mathbf{s}) + E_{\text{Temp}}(t, \mathbf{s}) \\ Y_{\text{Defo}}(t, \mathbf{s}) = \mu_{\text{Defo}}(t, \mathbf{s}) + X_{\text{Defo}}(t, \mathbf{s}) + E_{\text{Defo}}(t, \mathbf{s}) \end{cases}$$

- $\mu(t, \mathbf{s})$ the deterministic trend
- $X(t, \mathbf{s})$ the centered random residual
- $E(t, \mathbf{s})$ the measurement white noise

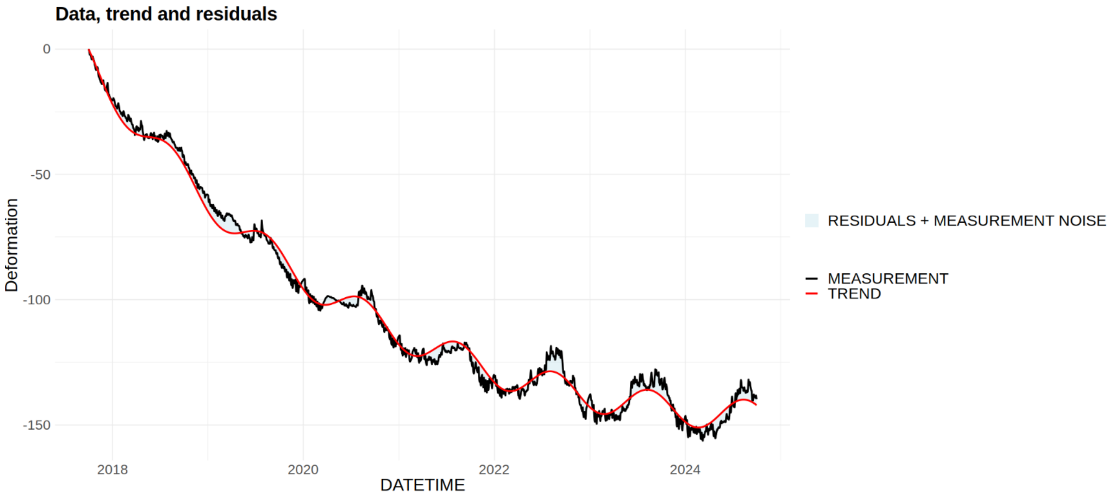
Questions :

- How to model the trend of each signal
- How to model the residuals

ILLUSTRATION SPATIO-TEMPORAL MODEL



■ ILLUSTRATION SPATIO-TEMPORAL MODEL



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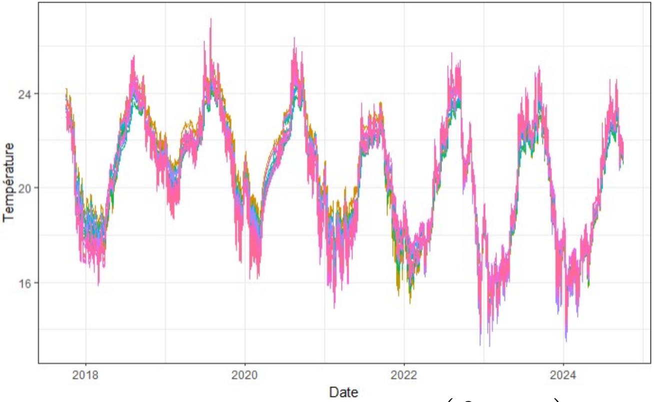
■ TREND MODELING FOR TEMPERATURE



$$\mu_{\text{Temp}}(t, \mathbf{s}) = \alpha_0(\mathbf{s}) + \alpha_1(\mathbf{s}) \sin\left(\frac{2\pi}{365} t + c\right)$$

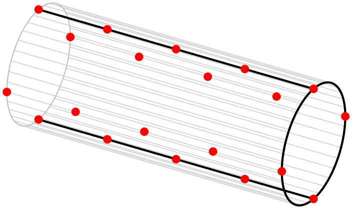
Similar for μ_{Defo} with coefficients β_j and exponential term

ESTIMATING THE COEFFICIENTS

Question : How to define functions $\beta_j(\mathbf{s})$ and $\alpha_j(\mathbf{s})$

Idea :

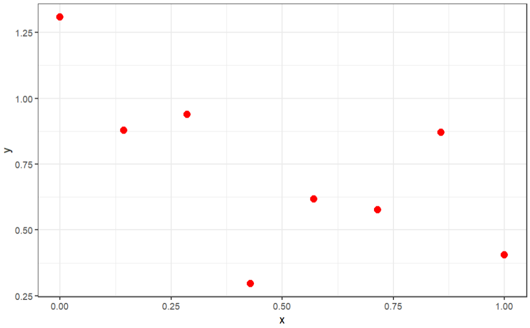
- Denote $(\mathbf{s}_i)_{i=1}^n$ the positions of the sensors
- Compute $\beta_j(\mathbf{s}_i)$ and $\alpha_j(\mathbf{s}_i)$, $1 \leq i \leq n$ with linear regression
- Use interpolation method to get $\beta_j(\mathbf{s})$ and $\alpha_j(\mathbf{s})$ on the surface of the cylinder

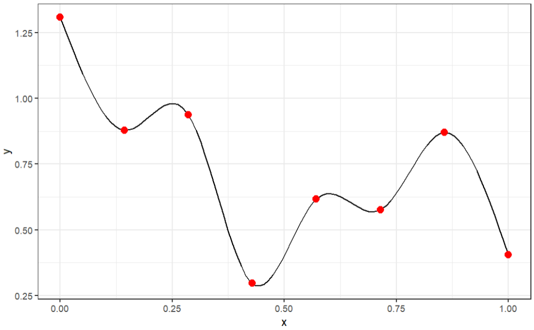


Specific study: Work with interpolating splines on Riemannian manifold

INTERPOLATION PROBLEM

Objective: Reconstruct an unknown function over the entire domain from n observations $\mathbf{y} = y_1, \ldots, y_n$ at points $\mathbf{s}_1, \ldots, \mathbf{s}_n$





■ SPLINE INTERPOLATION

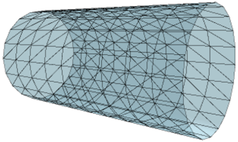
Idea : The interpolating spline minimizes an energy function $E(u)$ given the constraint

$$u(\mathbf{s}_i) = y_i,\ 1 \leq i \leq n$$

- Example in $\mathbb{R}$: $E(u) = \int_{\mathbb{R}} [u''(x)]^2\, dx$
- On manifold : $E(u) = \int_{\mathcal{M}} |\Delta u|^2 d\mu_g$

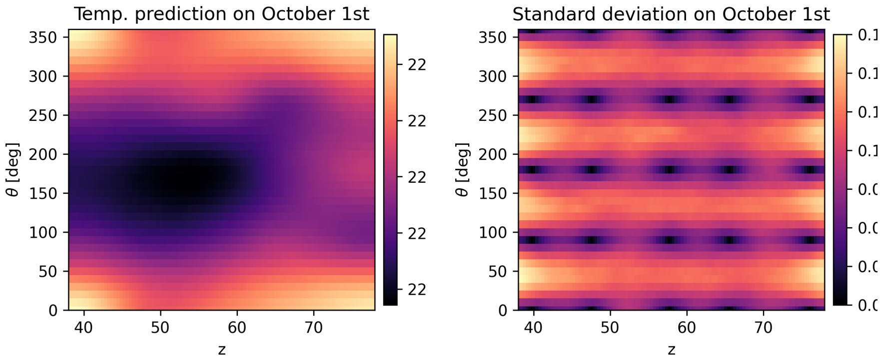
Problem: Some properties of Δ are not known on manifold

➡ Approximate the solution using a triangulation



ADVANTAGE OF THE APPROACH

- Very general : just need a triangulation of the manifold
- Deal with a large number of observations
- Account for uncertainties



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■ BACK TO THE SPATIO-TEMPORAL MODEL

$$\begin{cases} Y_{\text{Temp}}(t,\mathbf{s}) = \mu_{\text{Temp}}(t,\mathbf{s}) + X_{\text{Temp}}(t,\mathbf{s}) + E_{\text{Temp}}(t,\mathbf{s}) \\ Y_{\text{Defo}}(t,\mathbf{s}) = \mu_{\text{Defo}}(t,\mathbf{s}) + X_{\text{Defo}}(t,\mathbf{s}) + E_{\text{Defo}}(t,\mathbf{s}) \end{cases}$$

➡ Need to model the evolution of the couple $(X_{\text{Temp}}, X_{\text{Defo}})$

SPDE

SPDE = Stochastic Partial differential equation

Classically, the SPDE for a spatio-temporal random field $X(\mathbf{s}, t)$ has the form

$$\frac{\partial}{\partial t} X(t, \mathbf{s}) + P(-\Delta) X(\mathbf{s}, t) \; = \; Z(t, \mathbf{s}), \qquad s \in \mathcal{M}, \; t > 0$$

with $Z(t, \mathbf{s})$ a random spatio-temporal noise (not necessarily white) and P is a polynomial

Examples :

- Stochastic heat equation $\frac{\partial}{\partial t} X(t, \mathbf{s}) - \alpha \Delta X(\mathbf{s}, t) \; = \; Z(t, \mathbf{s})$
- Advection-diffusion $\left[\frac{\partial}{\partial t} + \frac{1}{c}\left(\kappa^2 - \nabla \cdot \mathbf{H} \nabla\right)^{\alpha} + \frac{1}{c} \gamma \cdot \nabla\right] X(t, \mathbf{s}) = \frac{\tau}{\sqrt{c}} Z(t, \mathbf{s})$

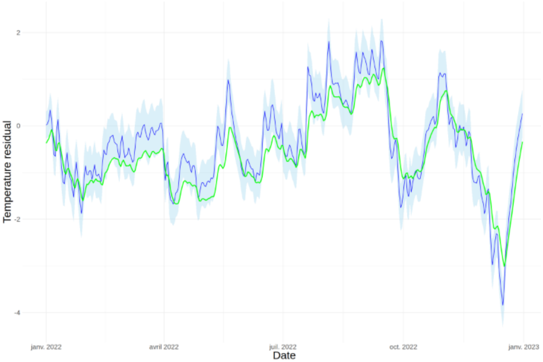
COUPLED SPDE

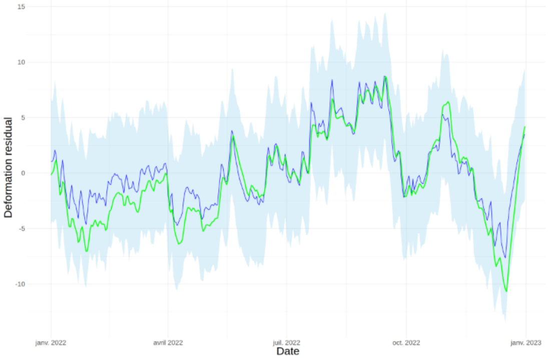
$$\begin{cases} \dfrac{\eta_1 \partial X_{\mathsf{Temp}}(\mathbf{s},t)}{\partial t} + P_{11}(-\Delta)\, X_{\mathsf{Temp}}(\mathbf{s},t) + P_{12}(-\Delta)\, X_{\mathsf{Defo}}(\mathbf{s},t) = Z_1(\mathbf{s},t), \\ \eta_2 \dfrac{\partial X_{\mathsf{Defo}}(\mathbf{s},t)}{\partial t} + P_{21}(-\Delta)\, X_{\mathsf{Temp}}(\mathbf{s},t) + P_{22}(-\Delta)\, X_{\mathsf{Defo}}(\mathbf{s},t) = Z_2(\mathbf{s},t). \end{cases}$$

where $P_{11}, P_{12}, P_{21}, P_{22}$ are polynomials, and η_1, η_2 are boolean

SPDE SOLUTION

Finite-element approximation of the solution using triangulation and finite-difference





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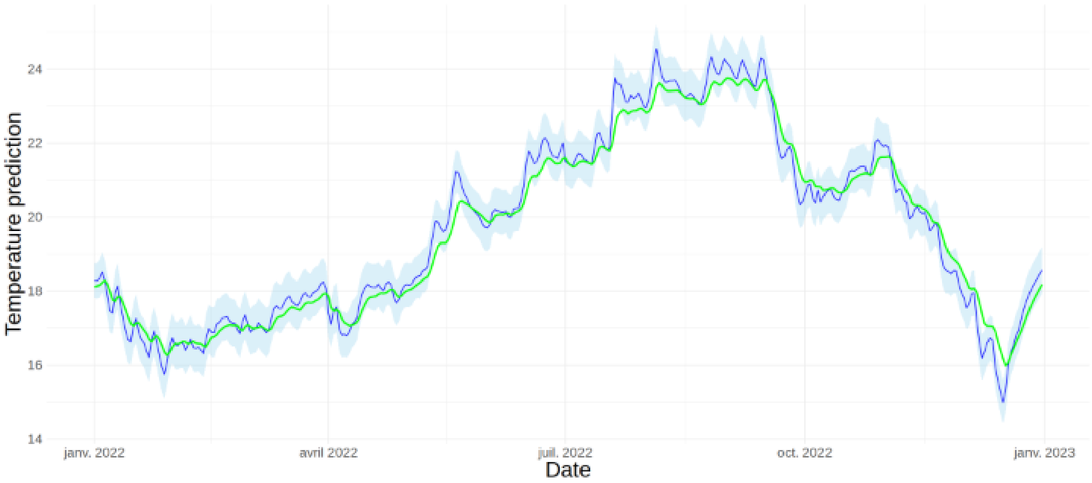
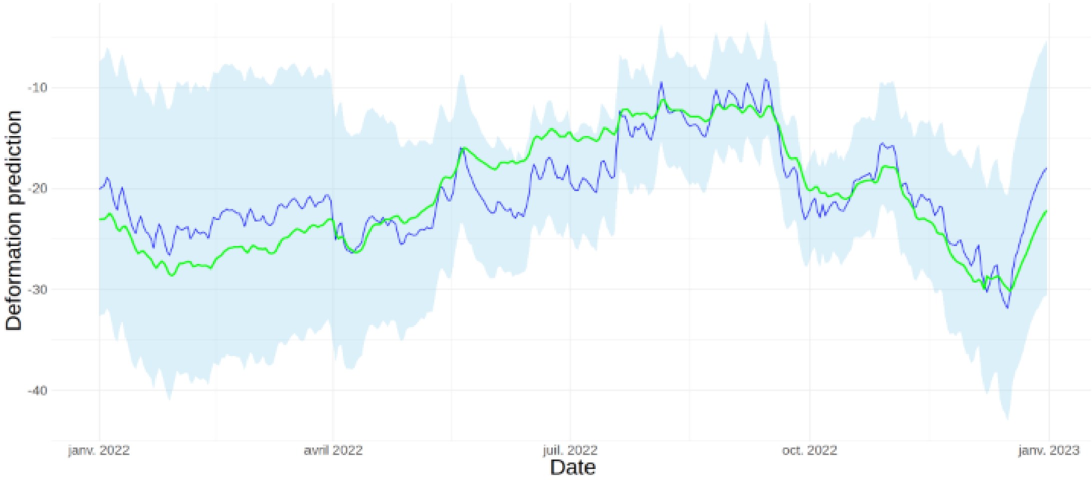
■ TEMPERATURE PREDICTION



DEFORMATION PREDICTION



ARTICLES

- Sire, C., Pereira, M., & Romary, T. (2025). **Spline Interpolation on Compact Riemannian Manifolds.** `https://hal.science/hal-05313523`
- Sire, C. & Pereira, M. (2026) **Uncertainty Quantification of Spline Predictors on Compact Riemannian Manifolds.** `https://hal.science/hal-05566721`

In preparation:

- Coupled system of spatio-temporal SPDEs
- Intrinsic Gaussian Random Fields for spatial prediction