Modeling daily precipitation occurrence, with long periods of drought

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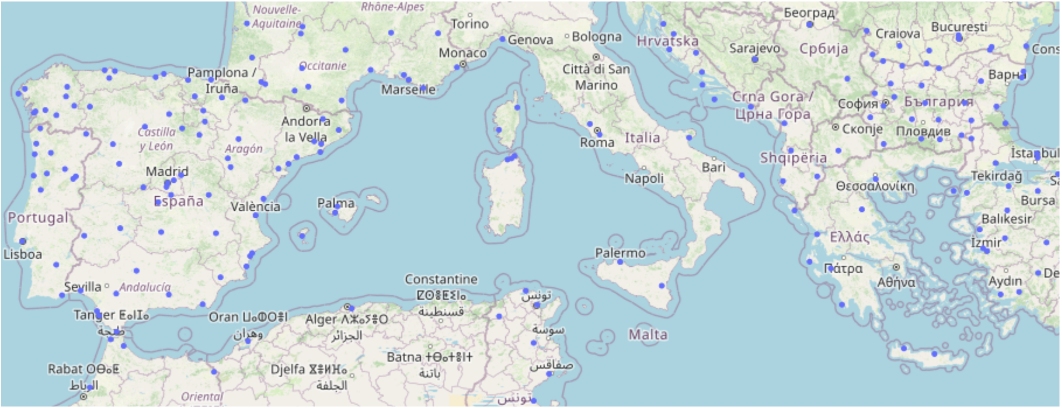
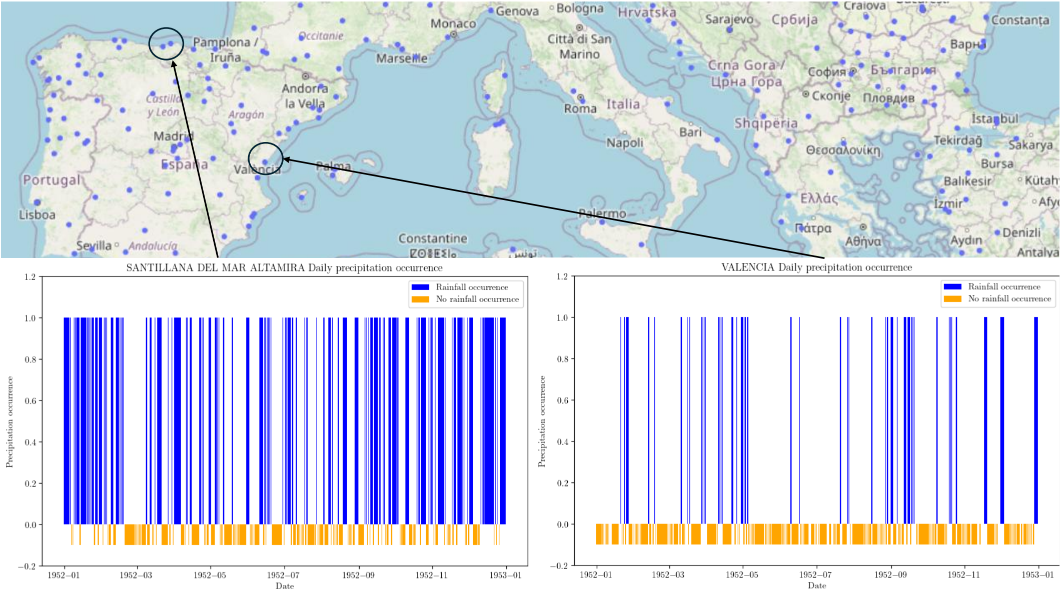
European Climate Assessment & Dataset

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SANTILLANA DEL MAR ALTAMIRA Daily precipitation occurrence
Rainfall occurrence
No rainfall occurrence
Precipitation occurrence
1.2
1.0
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Date
VALENCIA Daily precipitation occurrence
Rainfall occurrence
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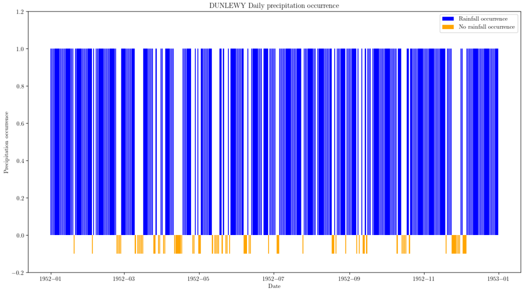
Date

Risk assessment requires several long scenarios

Recorded data



- Limitated to recorded duration ($\sim$ 10-50 years).
- No internal variability.
- No extrapolation beyond recorded values.

Stochastic Weather Generator

- Unlimited scenario length.
- Produce several scenarios (internal variability).
- Potential extreme extrapolation.

Markov chain point of view:

$$(R_n)_{n=1,2\dots},$$

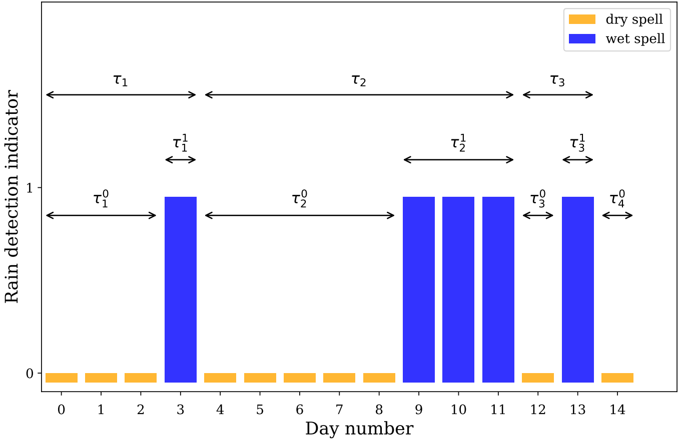
$R_n := \mathbb{1}_{\{\text{Rain has been recorded on day } n\}}$

Alternating Renewal model point of view:

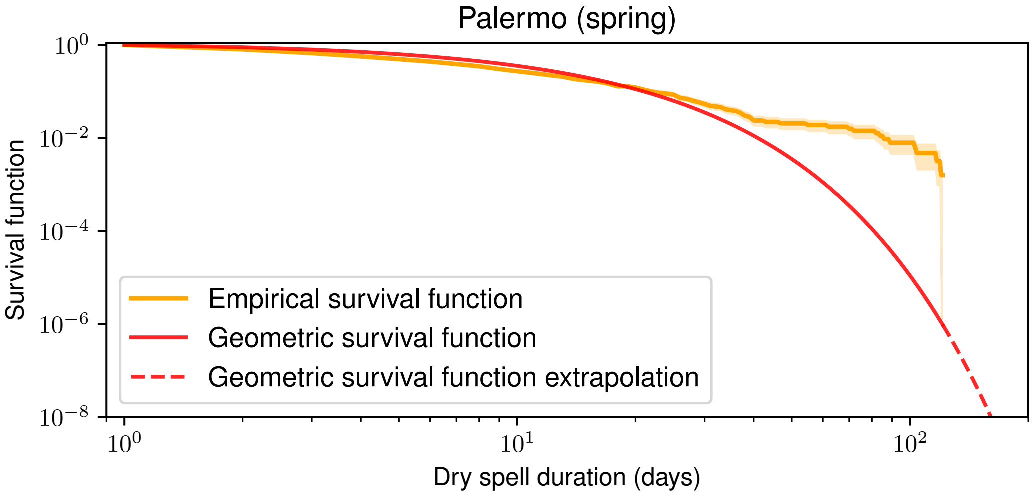
$$(\tau_k^{(0)}, \quad \tau_k^{(1)})_{k=1,2\dots},$$

$\tau_k^{(0)}$: duration of k^{th} dry spell, $\tau_k^{(1)}$: duration of k^{th} wet spell.

Binary Markov Chain with Duration is compatible with both points of view.

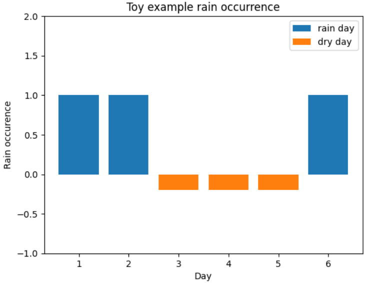


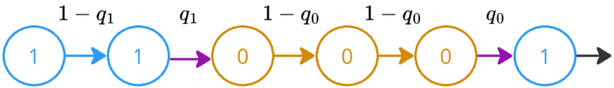
Survival function of $\tau^{(0)}$: empirical vs geometric fit



Modeling rainfall occurrence: intuition

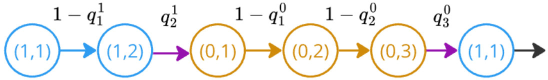
Rainfall occurrence toy data for 6 days:





two-states first-order Markov model $(R_n)_{n=0\dots}$

$$\mathbb{P}(\cdot) = \mathbb{P}_{(q_0,q_1)}(\cdot), \quad \mathbb{P}(\tau^{(0)} = d) = (1-q_0)^{d-1}\, q_0$$



Binary Markov Chain with Duration $(R_n, D_n)_{n=0\dots}$

$$\mathbb{P}(\cdot) = \mathbb{P}_{(q_d^{(0)},q_d^{(1)}),d=1,2\dots}(\cdot), \quad \mathbb{P}(\tau^{(0)} = d) = \left(\prod_{k=1}^{d-1} 1 - q_k^{(0)}\right) q_d^{(0)}$$

Binary Markov Chain with Duration (BMCD)

Let us have $\{q_d^{(r)}\}_{d\geq 1}$, $r = 0, 1$, sequences in $(0, 1)$.
For given initial values $r_0 \in \{0, 1\}$ and $d_0 \in \mathbb{N}$, set $(R_0, D_0) = (r_0, d_0)$, and for all $n \in \mathbb{N}$:

$$(R_{n+1}, D_{n+1}) = \begin{cases} (1 - R_n, 1), & \text{with probability } q_{D_n}^{(R_n)}, \\ (R_n, D_n + 1), & \text{with probability } 1 - q_{D_n}^{(R_n)}. \end{cases}$$

Link between BMCD and spell duration

Proposition (adapted from (Kozubowski)[1])

Let us have a distribution on $\tau^{(r)}$. Let us have a BMCD with parameters $\{q_d^{(r)}\}_{d\geq 1}$ given by:

$$q_d^{(r)} = \begin{cases} \mathbb{P}(\tau^{(r)} = d \mid \tau^{(r)} \geq d), & \text{if } \mathbb{P}(\tau^{(r)} \geq d) > 0, \\ 1, & \text{otherwise} \end{cases}, \forall\, d \geq 1.$$

This model has spell duration distributed as $\tau^{(r)}$.

- Choose a parametric distribution $\tau^{(0)}, \tau^{(1)}$.
- Estimate the parameters on $(\tau_k)_{k=1\ldots K}$.
- Retrieve the sequences $\{q_d^{(r)}\}_{d\geq 1}$.

[1] Tomasz J. Kozubowski, Dorota Młynarczyk, Anna K. Panorska "Waiting time representation of discrete distributions", Statistics and Probability Letters, 2025

$\tau^{(r)}$ distribution

$\tau^{(0)}$ **"hdeGP" distribution:** $\mathbb{P}_{f_1,\kappa,\sigma,\xi}(\tau^{(0)} = d) = \begin{cases} f_1, & d = 1, \\ (1 - f_1)\, F_{\kappa,\sigma,\xi}(d-1), & d \geq 2, \end{cases}$

with $F_{\kappa,\sigma,\xi}$ a discretized [2]type 1 extended Generalized Pareto Distribution (eGPD) probability mass function: TL; DR:

1. $\xi > 0$: heavy-tailed
2. $\xi \simeq 0$: exponential tail
3. $\xi < 0$: right-bounded tail

$\tau^{(1)}$ **mixt. geom. distribution:** $\mathbb{P}_{\pi,p_1,p_2}(\tau^{(1)} = d) = \pi\, p_1(1-p_1)^{d-1} + (1-\pi)\, p_2(1-p_2)^{d-1}.$

[2] P. Naveau, R. Huser, P. Ribereau, and A. Hannart. Modeling jointly low, moderate, and heavy rainfall intensities without a threshold selection. Water Resources Research, 52(4):2753–2769, 2016

Parameter estimation

$\tau^{(0)}$ parameters: $(f_1, \kappa, \sigma, \xi)$

$\widehat{f_1} = \frac{1}{n} \sum_{k=1}^{n} \mathbb{1}\{\tau_k^{(0)} = 1\}$.

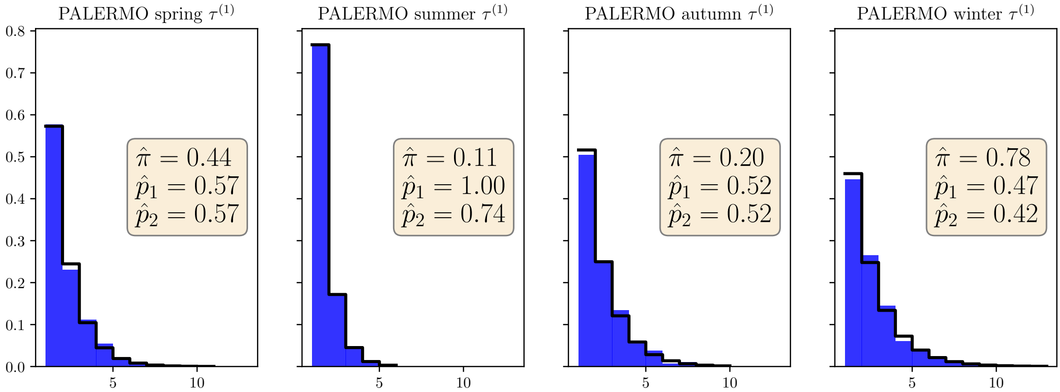
(κ, σ, ξ) estimated by [2]Probability Weighted Moments method

$\tau^{(1)}$ parameters: (π, p_1, p_2)

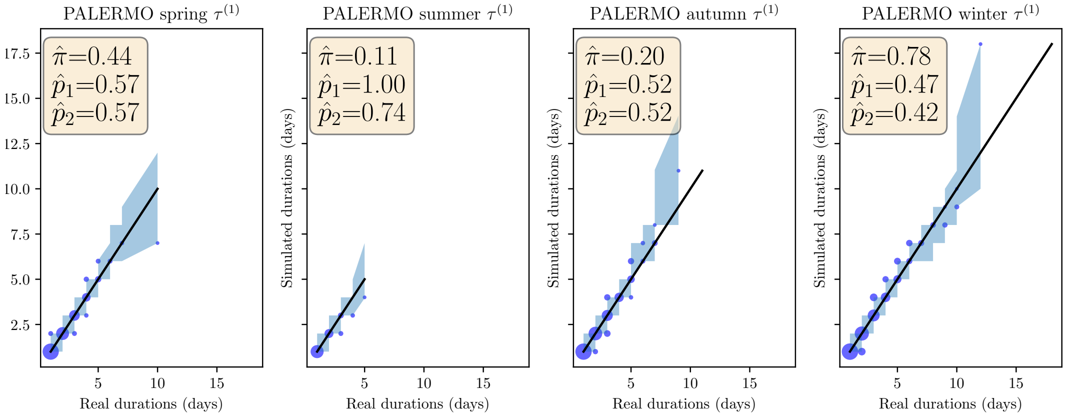
Estimated by Expectation-Maximization algorithm.

[2] P. Naveau, R. Huser, P. Ribereau, and A. Hannart. Modeling jointly low, moderate, and heavy rainfall intensities without a threshold selection. Water Resources Research, 52(4):2753–2769, 2016

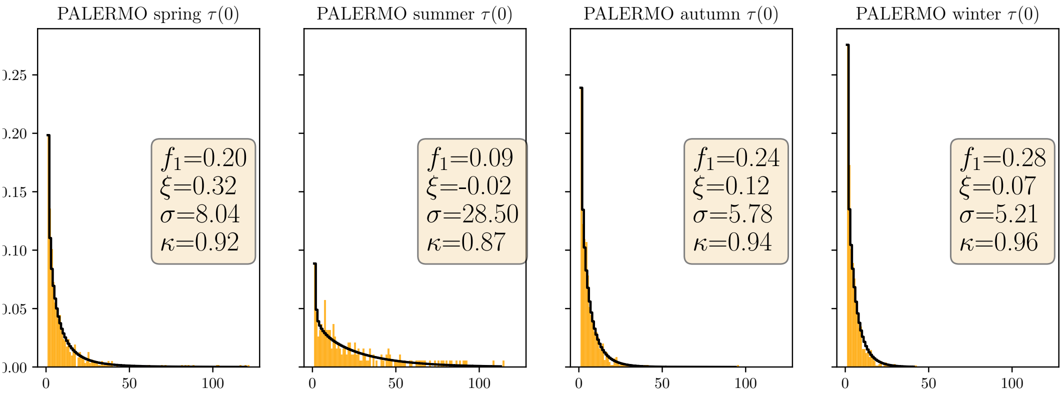
Histogram wet spell duration



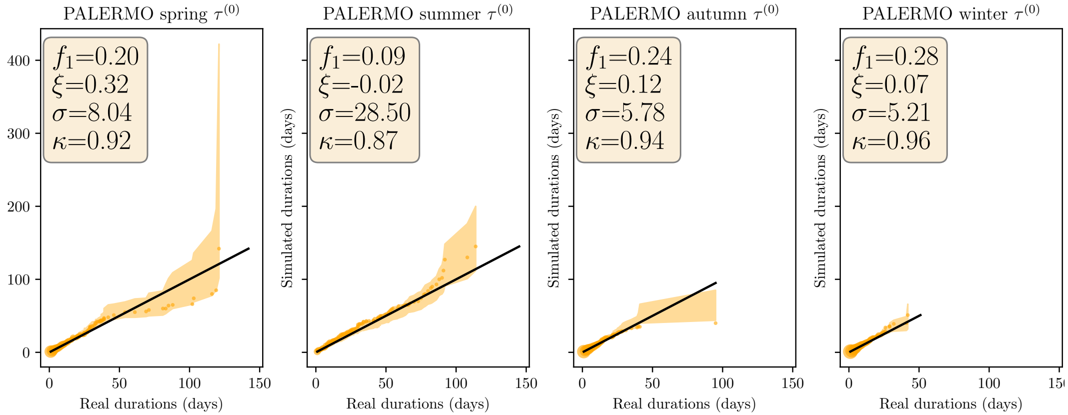
QQplots wet spell duration



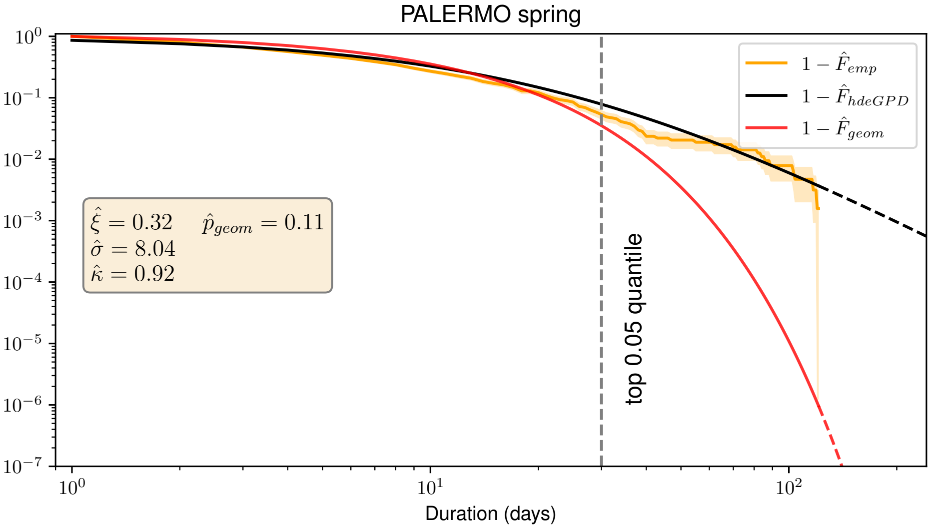
Histogram dry spell duration



QQplots dry spell duration



Survival function of dry spell duration for Palermo spring ($\hat{\xi} = 0.32$)



Consequence: Share of Time Spent in Severe Dry Spells

Main result

Our representation gives asymptotic results on quantities such as:

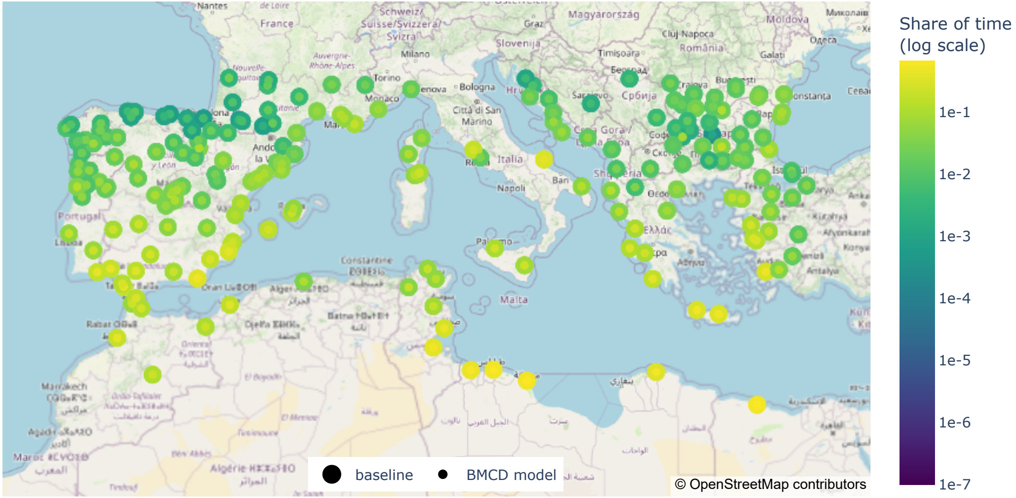
$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=0}^{n} \mathbb{1}_{\{R_k=0,\ D_k\geq d\}} = \frac{\mathbb{E}\left[\max\left(0, \tau^{(0)} - d\right)\right]}{\mathbb{E}[\tau]} \qquad \text{a.s.}$$

Interpretation: this is the long-run share of time spent in dry spells of length at least d.

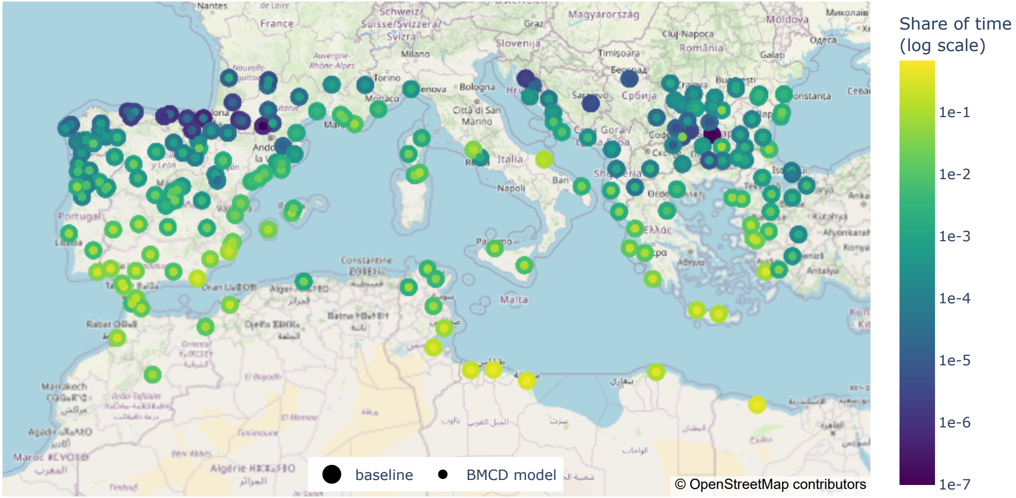
Comparison:

- geometric model,
- our model.

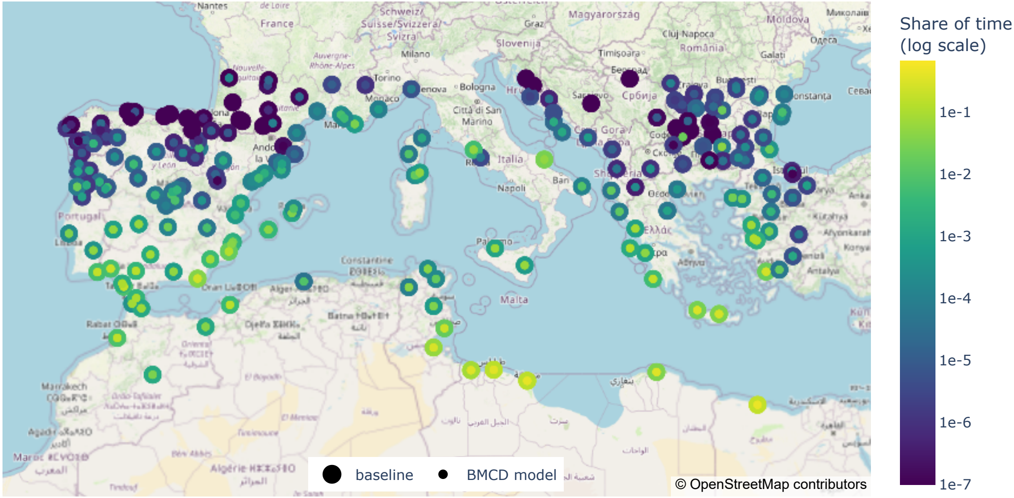
Dry spell share ≥ 20 days



Dry spell share ≥ 40 days



Dry spell share ≥ 60 days



Spatial extension of the model [3]

Single site model:

$$(R_{n+1}, D_{n+1}) = \begin{cases} (1 - R_n, 1), & \text{w. prob. } q^{(R_n)}(D_n), \\ (R_n, D_n + 1), & \text{w. prob. } 1 - q^{(R_n)}(D_n). \end{cases}$$

Spatial extension, define for any $\mathbf{s}$ in a domain $\mathcal{D}$:

$$Z_n(\mathbf{s}) = (-1)^{R_n(\mathbf{s})}\, Y_n(\mathbf{s}), \quad z_n(\mathbf{s}) = \Phi^{-1}\left[q^{(R_n(\mathbf{s}))}(D_n(\mathbf{s}))\right],$$

with $(Y_n)_{n\geq 1}$ i.i.d. Gaussian processes on $\mathcal{D}$, having covariance functions $(C_n)_{n\geq 1}$, and Φ the c.d.f. of a standard Gaussian.

$$(R_{n+1}(\mathbf{s}), D_{n+1}(\mathbf{s})) = \begin{cases} (1 - R_n(\mathbf{s}),\ 1), & \text{if } Z_n(\mathbf{s}) \leq z_n(\mathbf{s}), \\ (R_n(\mathbf{s}),\ D_n(\mathbf{s}) + 1), & \text{if } Z_n(\mathbf{s}) > z_n(\mathbf{s}). \end{cases}$$

[3] D. S. Wilks. Multisite generalization of a daily stochastic precipitation generation model. Journal of Hydrology, 210(1-4): 178–191, 1998. doi: 10.1016/S0022-1694(98)00186-3.

Conclusion

1. Binary Markov Chain with Duration: (Markov chain / alternating renewal model).
2. Extreme value theory distributions: severe dry spells.
3. "Easy" spatialization.

Thank you for your listening !

[1] Richardson, C. "Stochastic simulation of daily precipitation, temperature, and solar radiation", Water Resources Research, 1981

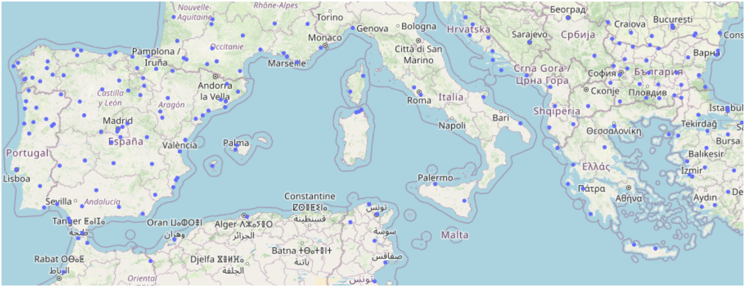
[2] Tomasz J. Kozubowski, Dorota Młynarczyk, Anna K. Panorska "Waiting time representation of discrete distributions", Statistics and Probability Letters, 2025

[3] Naveau, P., R. Huser, P. Ribereau, and A. Hannart, Modeling jointly low, moderate, and heavy rainfall intensities without a threshold selection, Water Resour. Res., 52, 2753-2769, 2016

[4] Ailliot, P., Allard, D., et al. "Stochastic weather generators: an overview of weather type models". Journal de la société française de statistique, 156(1), 101-113, 2015

[5] S. I. Resnick. "Adventures in Stochastic Processes". Birkhäuser Boston, 1992.

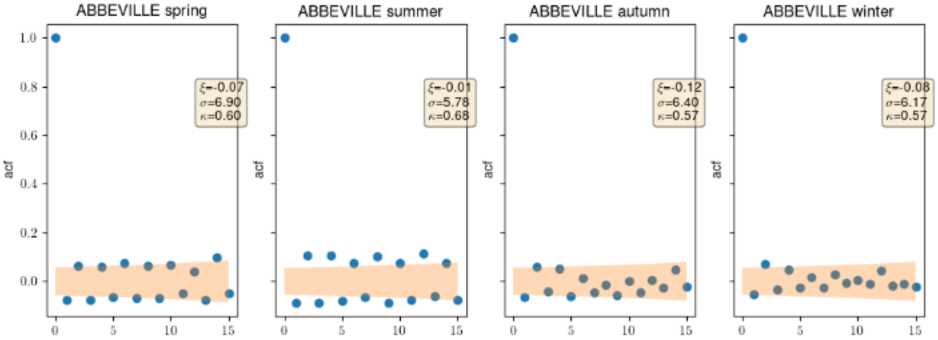
European Climate Assessment & Dataset: data processing



1. South Europe (*lat* < 45) stations daily rainfall records.
2. Only keep stations with more than 50 years of data since 1945, less than 5% missing values.
3. Dropped incomplete spells when missing values.
4. Daily rainfall recordings ≤ 0.6mm are considered as dry.
5. Filter out stations to have a constant "station density".
6. $\sim$200 stations are considered.

Check modeling hypothesis

In an altenating renewal model, we suppose mutual independance of the $(\tau_k^{(r)})_{k=1\dots}$. We check the autocorrelation.



Condition on the sequence $(q_d^{(r)})_{d=1\dots}$

$$\tau_1^{(0)} < \infty \;\; a.s.$$

if and only if

$$\sum_{d=1}^{\infty} q_d^{(0)} = \infty.$$

We consider this condition in order for the alternating renewal chain modeling to be relevant.

Consequence of waiting time representation

Using enlarged state space (R_n, D_n), let us control spell length distribution.

1. If $\tau^{(r)}$ has geometric distribution, $q_d^{(r)} := q \in (0, 1)$.
2. If $\tau^{(r)}$ has discrete Weibull distribution, $q_d^{(r)} := 1 - \exp(-\lambda(d+1)^\beta - d^\beta)$.
3. If $\tau^{(r)}$ has discrete Pareto distribution, $q_d^{(r)} := 1 - (\frac{1+\sigma\alpha d}{1+\sigma\alpha(d+1)})^{1/\alpha}$.
4. If $\tau^{(r)}$ has discrete extended-GPD distribution, $q_d^{(r)} := \frac{G(H(\frac{d+1}{\sigma})) - G(H(\frac{d}{\sigma}))}{1 - G(H(\frac{d}{\sigma}))}$ (with given G and H, details in appendix).

Coupling dry spell wet spell spatial