A few statistical challenges in glaciology

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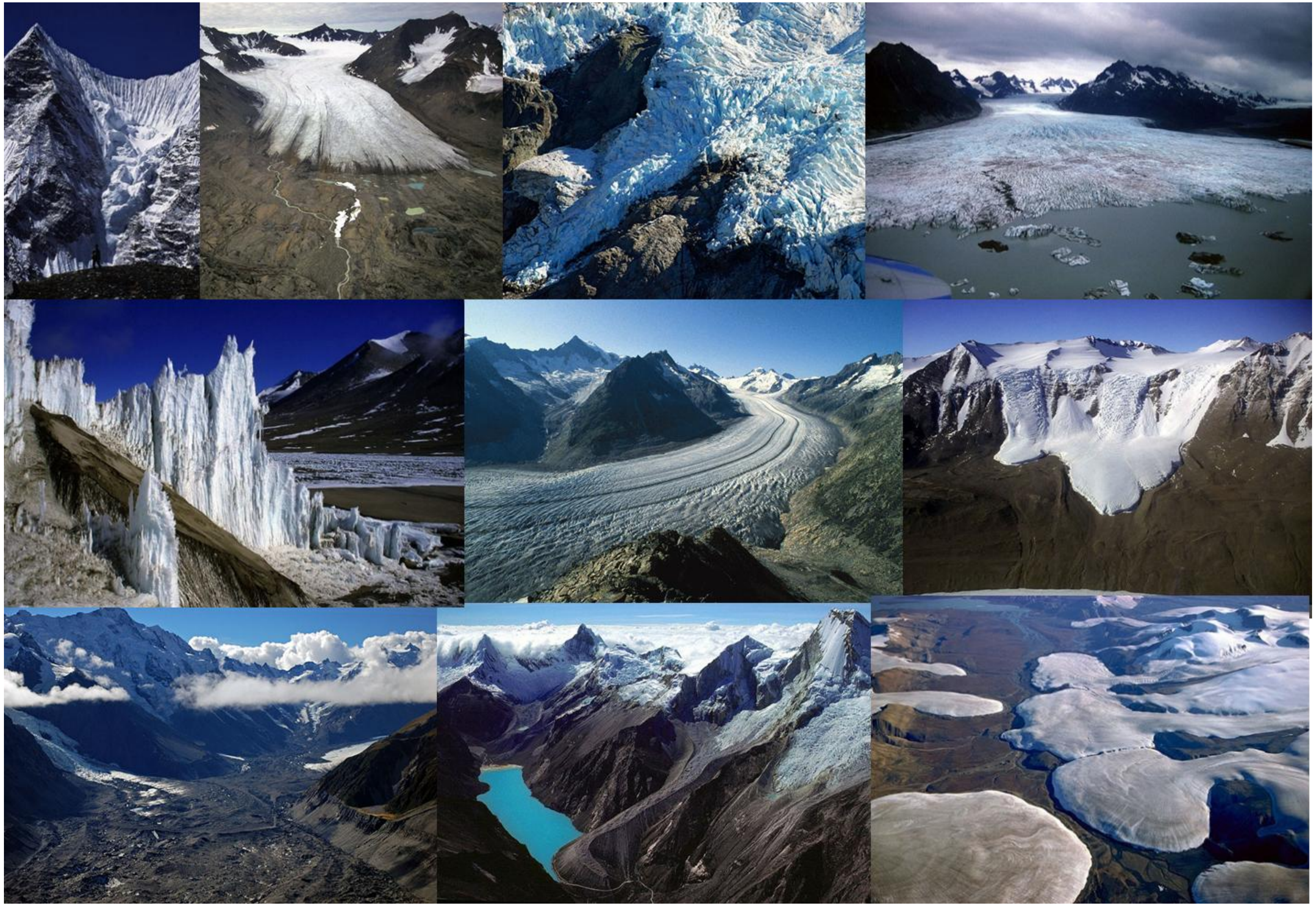
GEOLEARNING
CHAIRE /// Data Science for the Environment

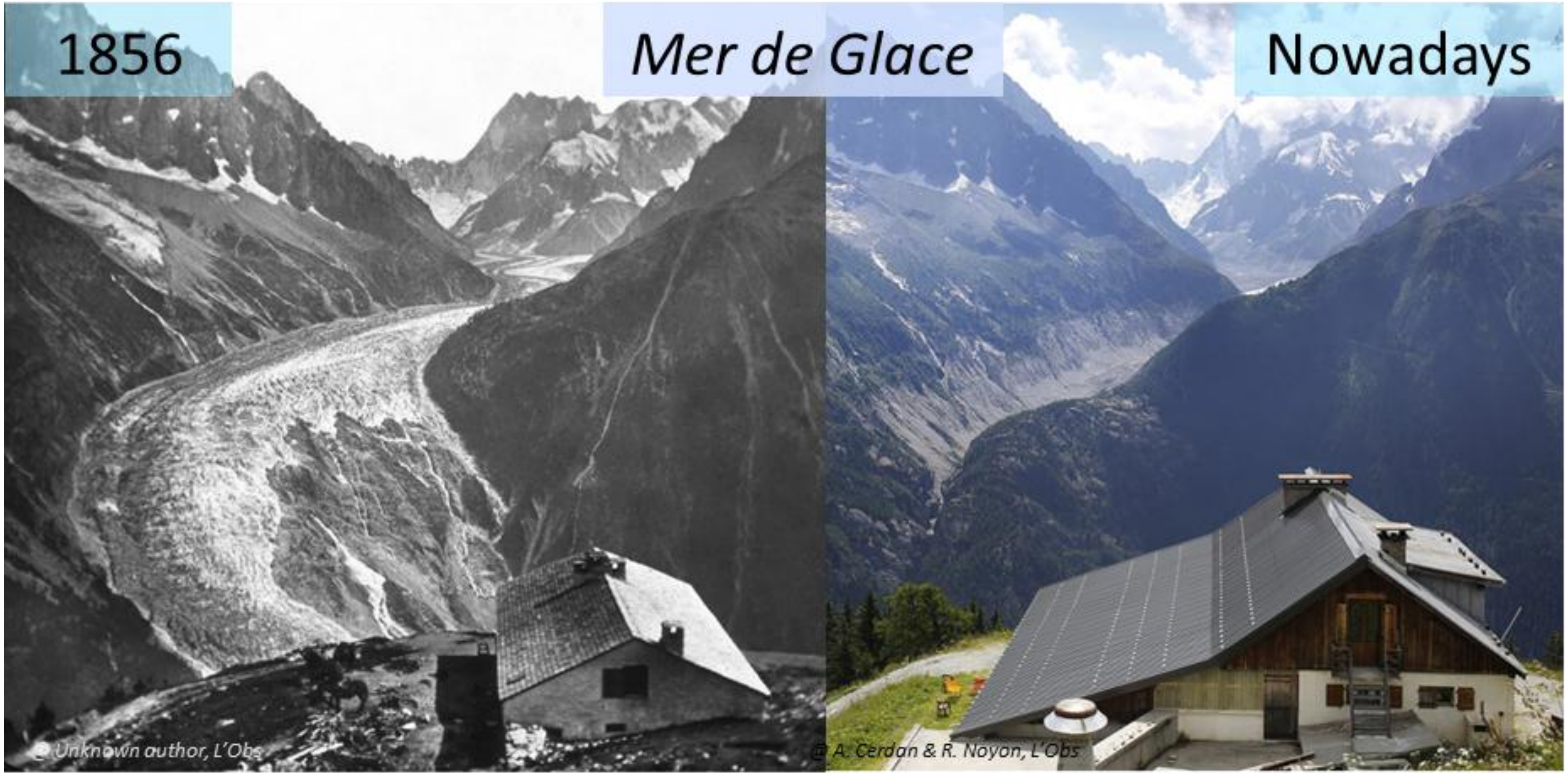
Geolearning, Fréjus, 31 march 2025

Today’s menu :

- **Context**
- Glacier shrinkage: from local to global scale
- Conclusions

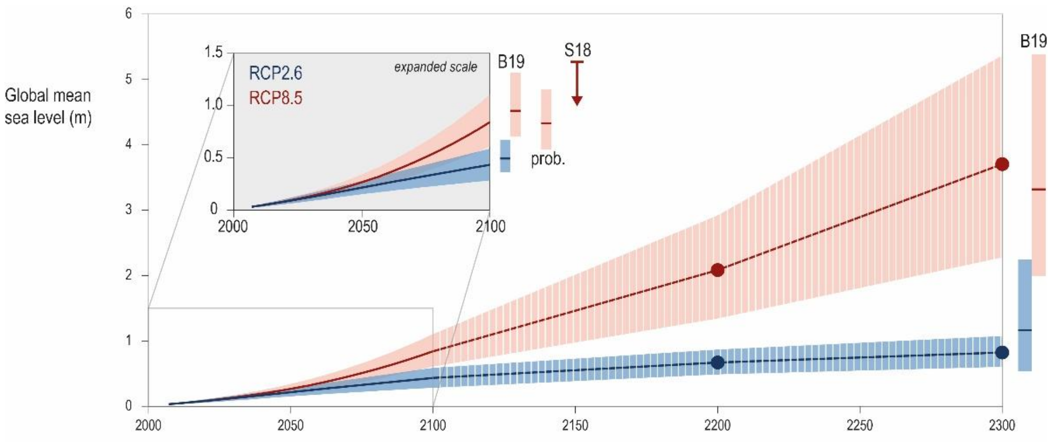


Context (1) : climate warming most iconic representation



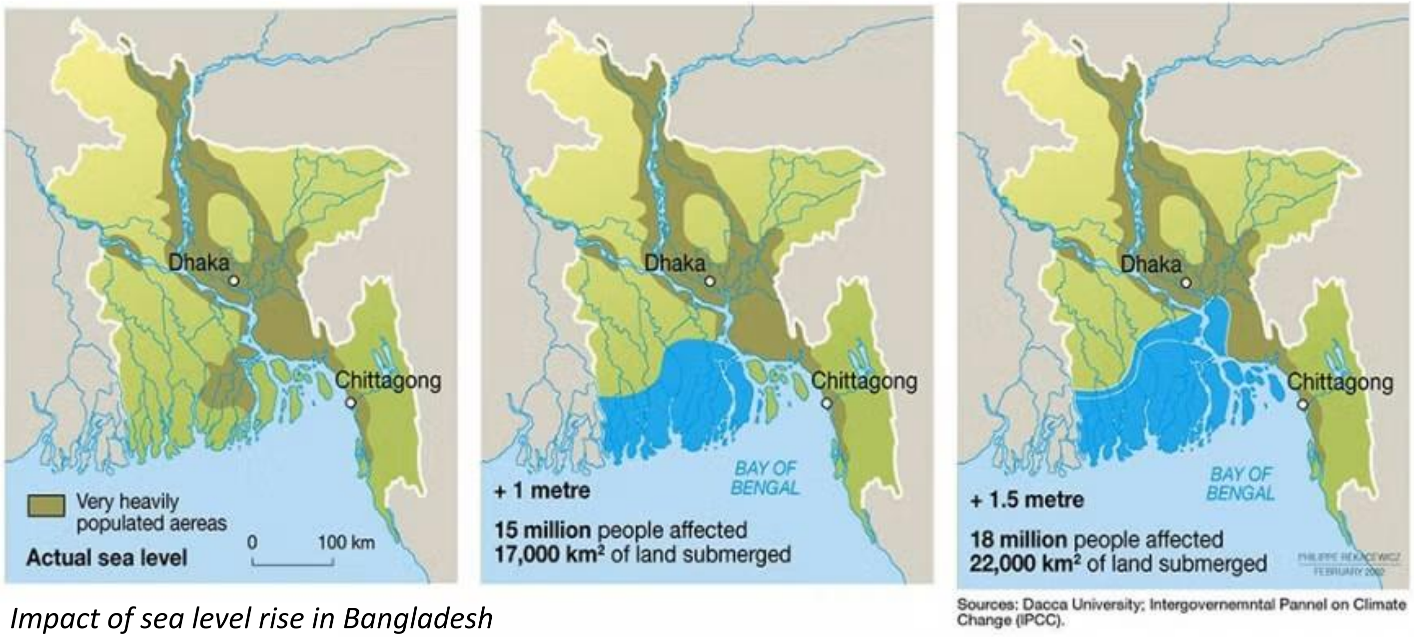
From Little Ice Age to the Age Without Ice…

Context (2) : sea level rise and consequences



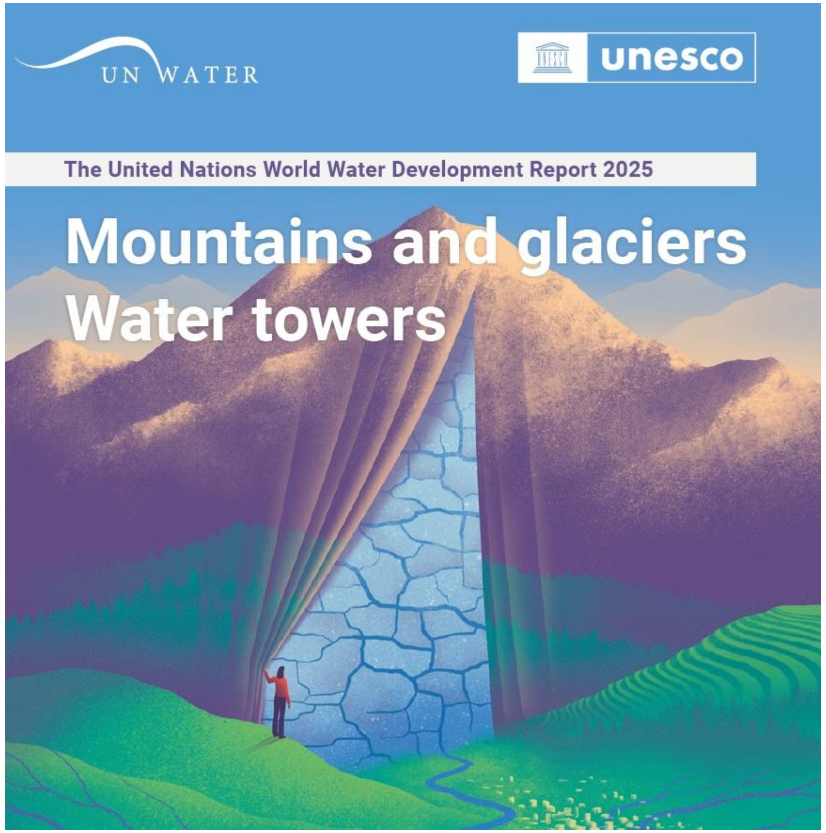
Projected sea level rise, IPCC (2019)

Irish coast during storm, @Irish Independent



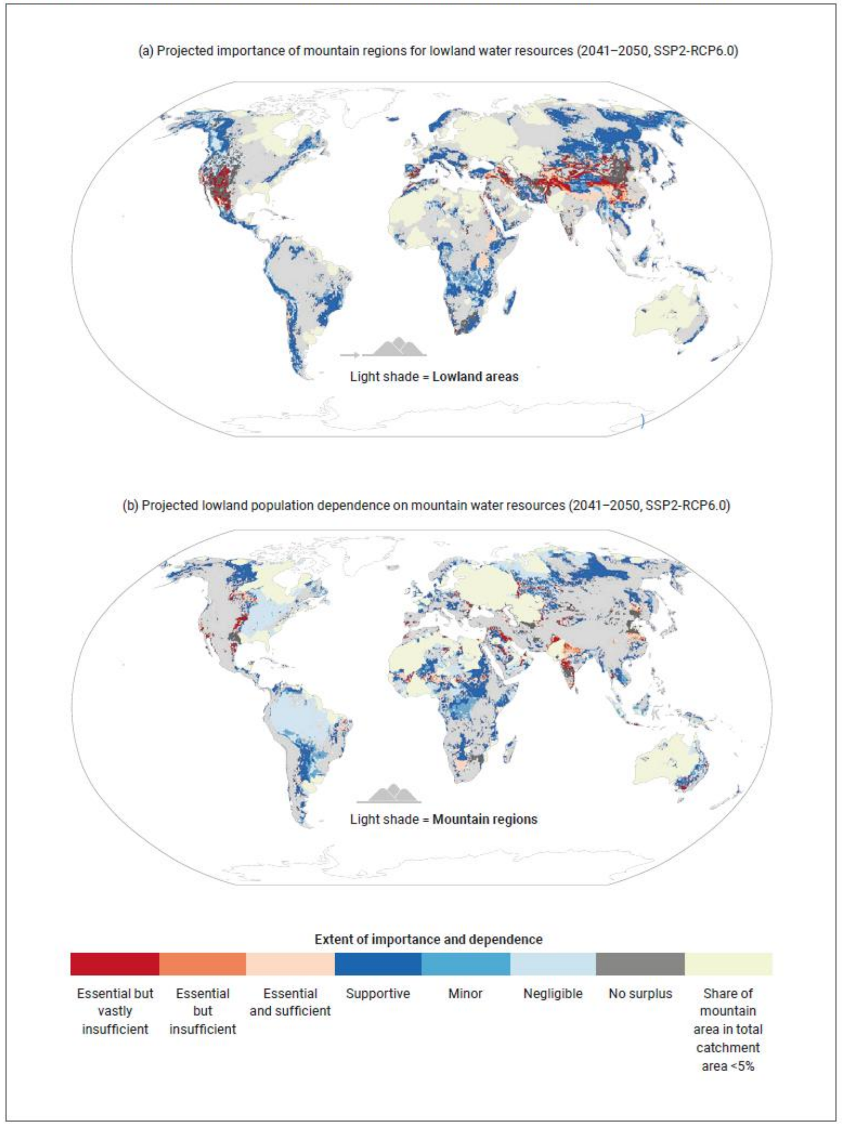
Impact of sea level rise in Bangladesh

Context (3) : changes in hydrological regimes and consequences



- Gradual change in seasonality of streamflows
- Peak regime before disappearance of glacial component of the water budget
- 2 billions of people affected (debated)

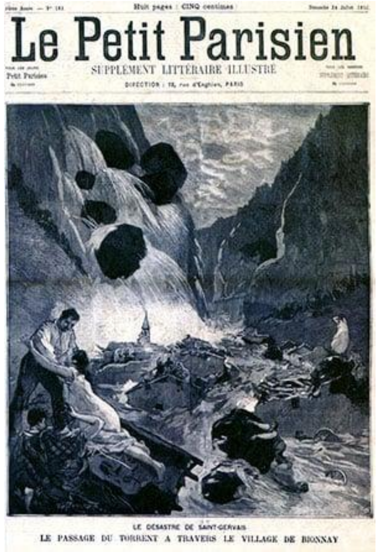
Figure 1.2 Projected importance of mountain regions and population dependence, 2041–2050 (SSP2-RCP6.0)



Source: Adler et al. (2022, fig. CCP5.2(a, b), p. 2282).

Context (4) : transient disaster risk

- Multiplicity of processes / risks: icefall, GLOFs, debris flows and rockfall involving fresh sediments, complex combinations, etc.
- Extreme non-stationarity
- Locally very rare but catastrophic, with far-reaching consequences



GLOF in Saint Gervais, 1892, (175 casualties)

Chamoli (India) burst flood, 2021 : 50-200 casualties

La Berarde, 2024. Picture @ONF

A few challenges…

- Ice budget, streamflows and sea level rise: spatio-temporal assessment and impacts
- Disaster risk: spatio-temporal prediction, anticipation
- Combination of data sources and scales, seamless modelling chains, uncertainty propagation, etc.

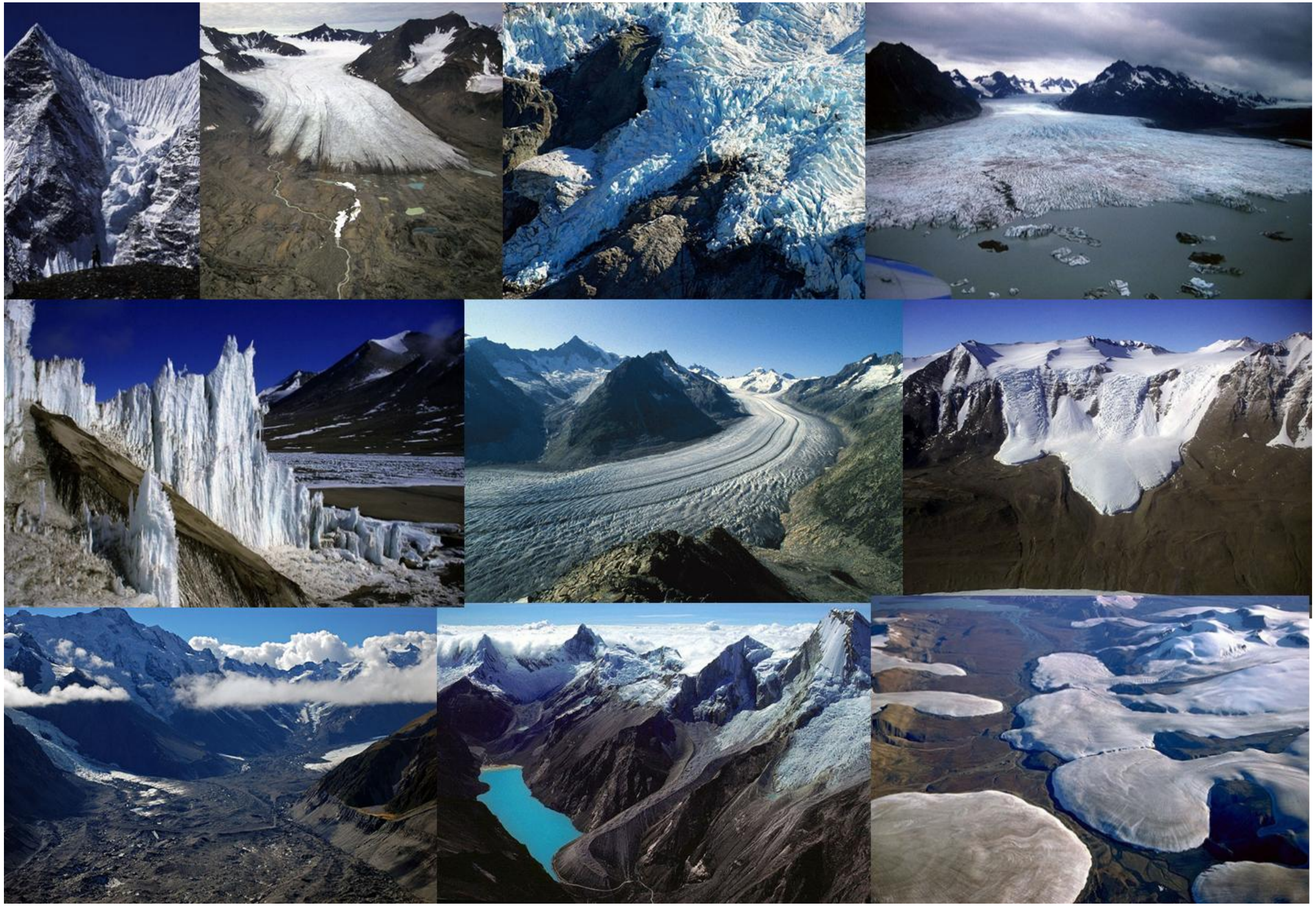
Potential glacier collapse (Aosta Valley, 2020) and evacuated area

Today's menu :

- Context

- **Glacier shrinkage: from local to global scale**

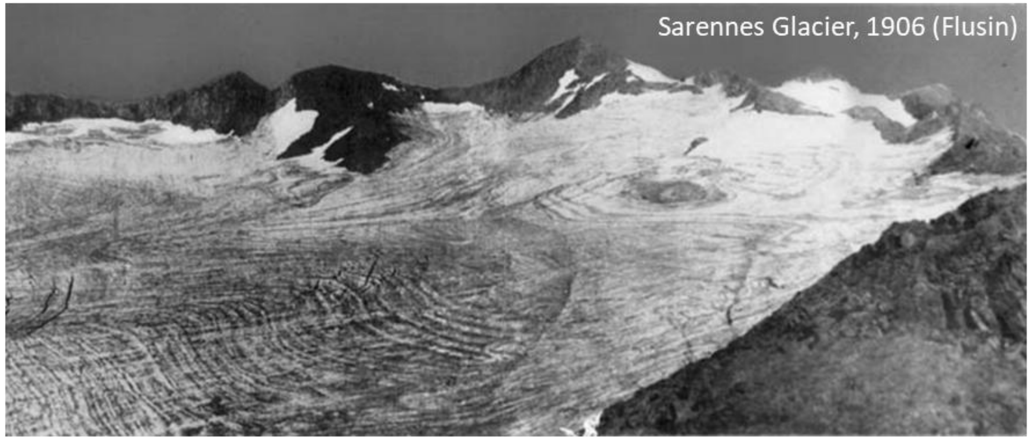
- Conclusions



The sad story of Sarennes glacier

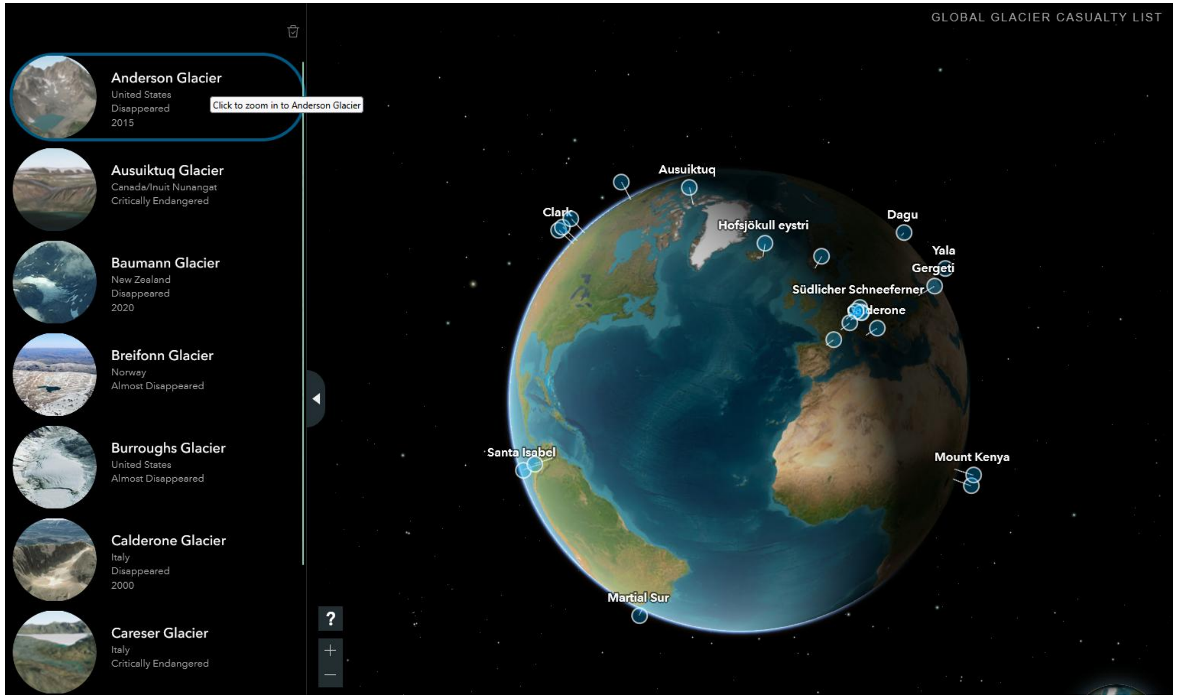
- Sarennes exceptional series:

- One of the world's oldest with continuous records of both winter and summer balance (1949-2022)
- Almost total disappearance: will integrate the official list of glacier casualties soon



Sarennes Glacier, 1906 (Flusin)

Sarennes Glacier and sad glaciologists, 2021 (@Dauphiné Libéré)

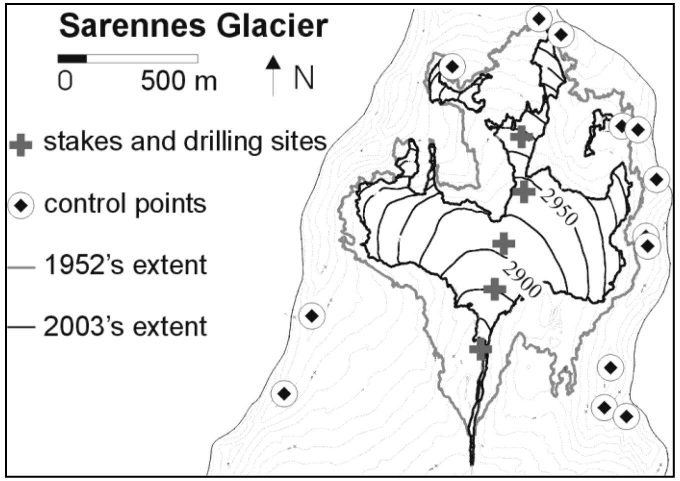


Extracting the local temporal signal

- Lliboutry (1974) approach:

$$b_{it} = a_o + \alpha_i + \beta_t + \varepsilon_{it}$$
$$\sum_i \alpha_i = \sum_t \beta_t = 0$$
$$\varepsilon_{it} \sim N\left(0, \sigma^2\right)$$

✓ Spatial-temporal variance decomposition of annual balance b
✓ An annual value free from local and geometric effects.



- A bivariate physically-oriented change-point approach:

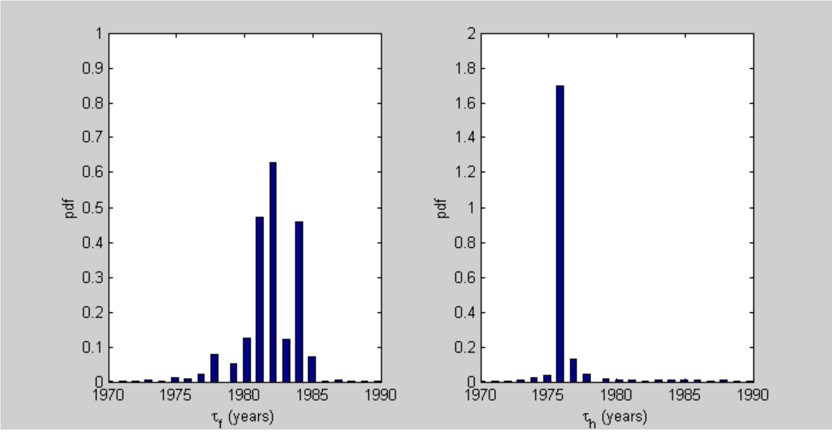
$$\begin{pmatrix} b_{it} \\ h_{it} \end{pmatrix} \sim N_2\left(\begin{pmatrix} \alpha_{bi} + \beta_{bt} \\ \alpha_{hi} + \beta_{ht} \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right) \quad f_{it} = h_{it} - b_{it}$$

$$\beta_{ft} \sim N\left(a_{f2} + b_{f2}t, \sigma^2_{f2}\right), t \in \left[\tau_f + 1, t_o + T - 1\right]$$
$$\beta_{ft} \sim N\left(a_{f1} + b_{f1}t, \sigma^2_{f1}\right), t \in \left[t_o, \tau_f\right]$$

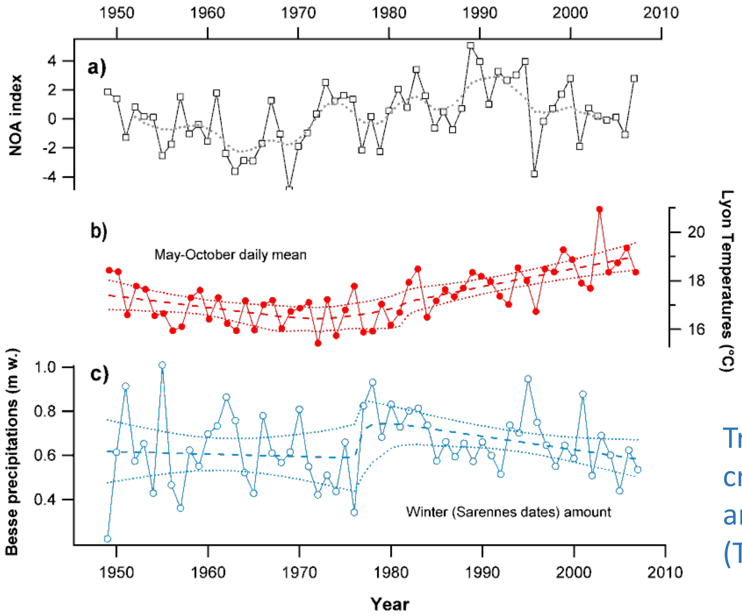
$$\beta_{ht} \sim N\left(a_{h2} + b_{h2}t, \sigma^2_{\beta h2}\right), t \in \left[\tau_h + 1, t_o + T - 1\right]$$
$$\beta_{ht} \sim N\left(a_{h1} + b_{h1}t, \sigma^2_{\beta h1}\right), t \in \left[t_o, \tau_h\right]$$

✓ Observation model on measured winter balance h and annual balance b
✓ Change point models for the two seasonal components h and f
✓ Mass balance trend: three linear segments/variances and two change points

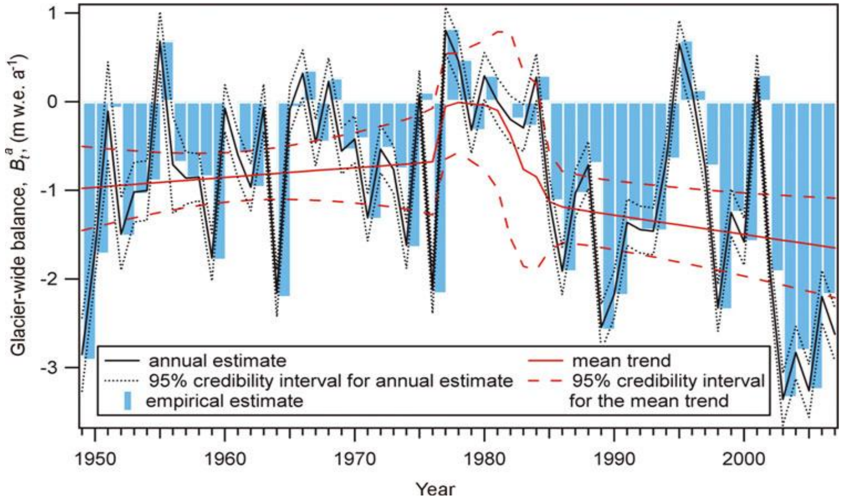
Results and climate relevance



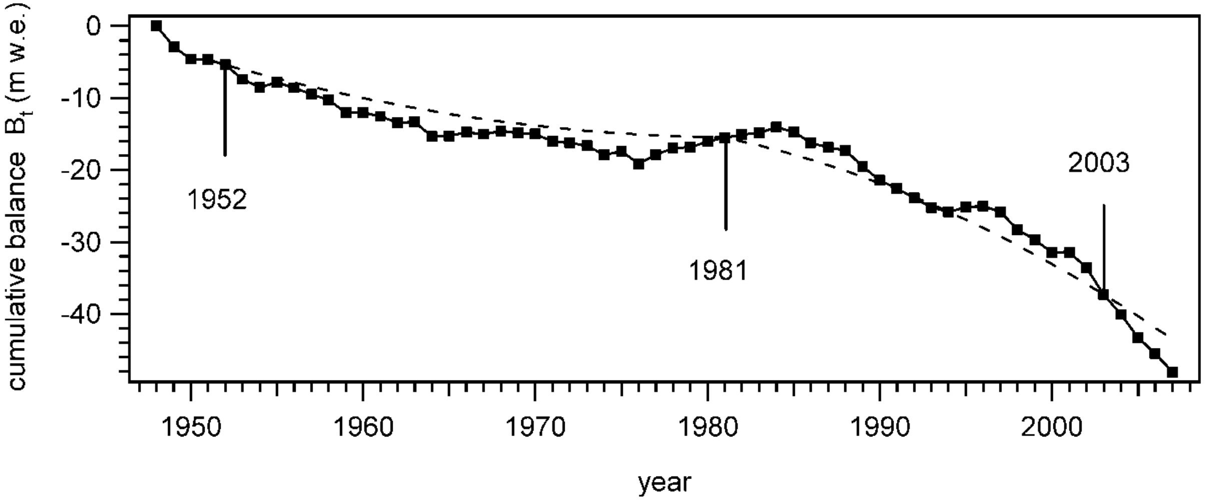
Posterior distribution of two well marked but distinct change points in ablation/accumulation (Eckert et al., 2011)



Trends and associated 95% credible intervals in local and synoptic covariates (Thibert et al., 2013).



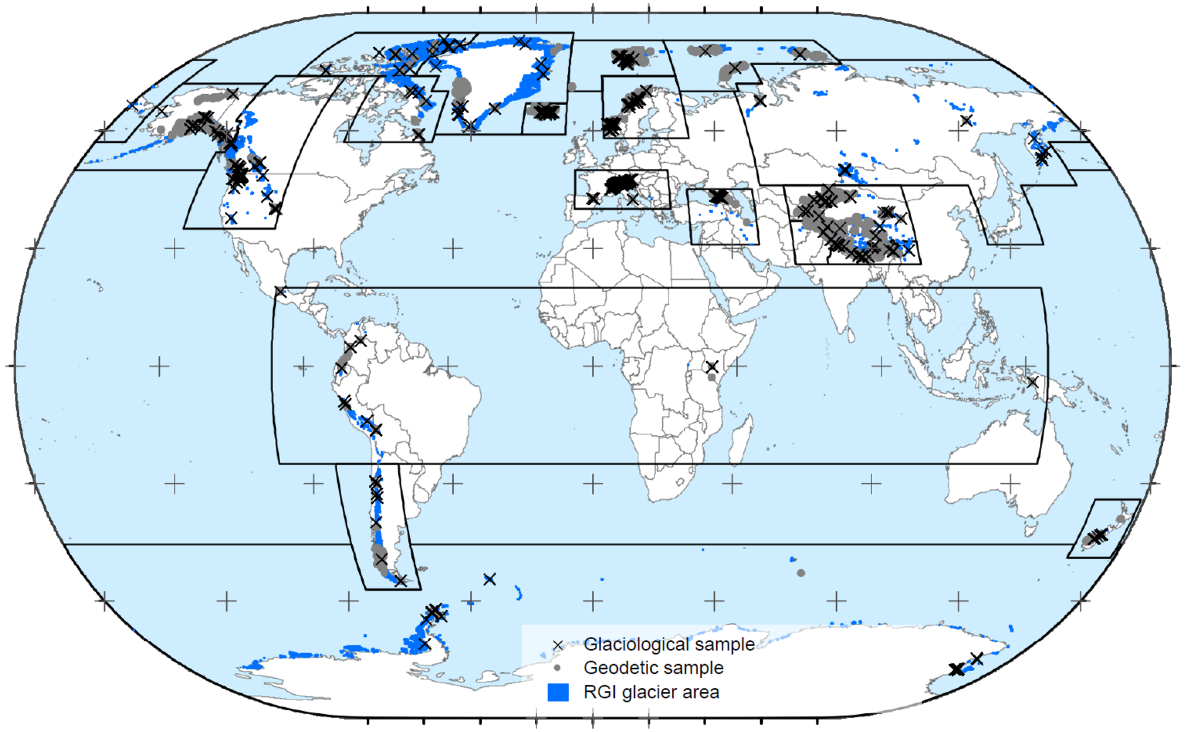
Annual balance (Eckert et al., 2011)



Cumulative balance (total of mass loss since 1949) with segmented parabolic shape due to underlying trends (Eckert et al., 2011):

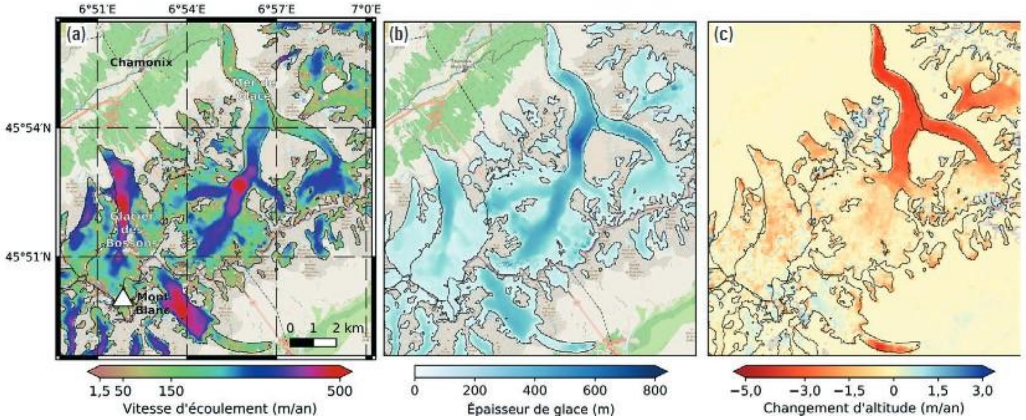
From local to global scale: numerous but diverse data

- In 2019, 215,000 glaciers distinct from the Greenland & Antarctic Ice Sheets covering 158,000 km^2
- 19 groups / regions
- Few pointwise measurements of uneven length versus extensive remote sensing coverage over the last years/decades



2019 WGMS glacier inventory

Pointwise mass balance measurements
© E. Thibert, INRAE.



Theia glacier atlas (Mer de glace, France)

Temporal signal for each group from measurements (1)

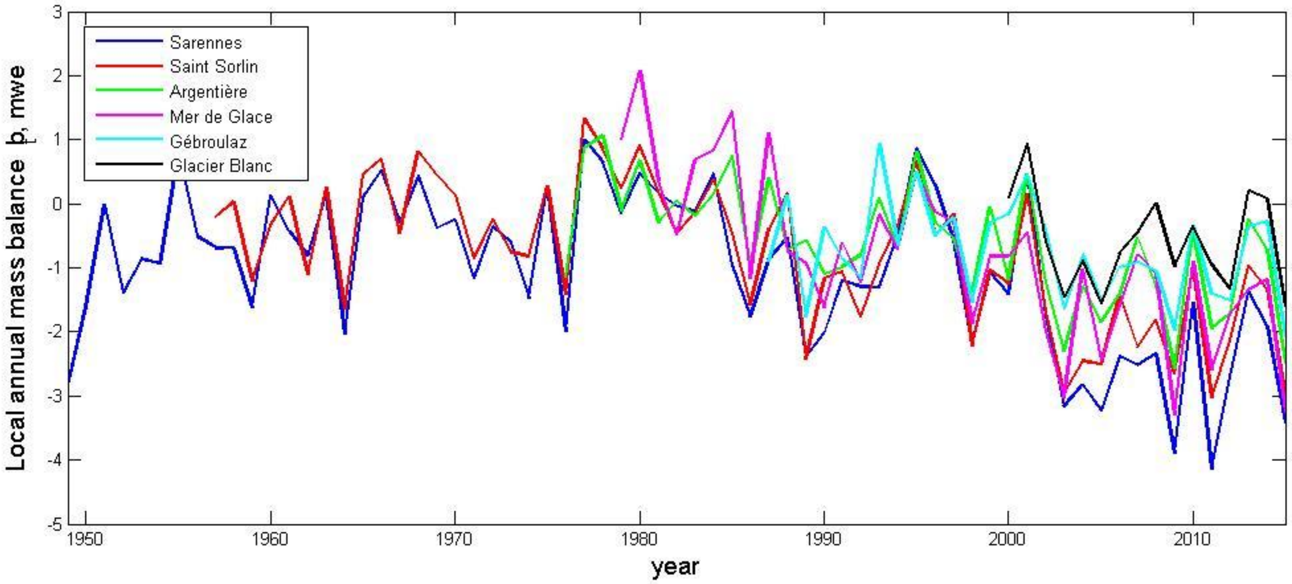
- A regional scale formulation of the variance decomposition with a smooth temporal structure

$$b_{it} = a_o + \alpha_i + \beta_t + \varepsilon_{it}$$

$$\beta_t = g_t + z_t \quad \text{with} \quad z_t \sim N\left(0, \sigma_z^2\right)$$

$$p(g) = \frac{|A|_+^{1/2}}{\delta_1^{1/2(T-2)}} \exp\left(\frac{-1}{2\delta_1} g' A g\right)$$ Wahba (1978)

$$\underbrace{\mathrm{p}(\theta, x | y)}_{\text{Joint posterior of model unknowns}} \propto \underbrace{\pi(\theta)}_{\text{Prior}} \times \underbrace{\mathrm{p}(y | \theta, x)}_{\text{Likelihood}} \times \underbrace{\mathrm{p}(x | y, \theta)}_{\text{Latent/process variables (« filtering density »)}}$$



All measurements available in France!

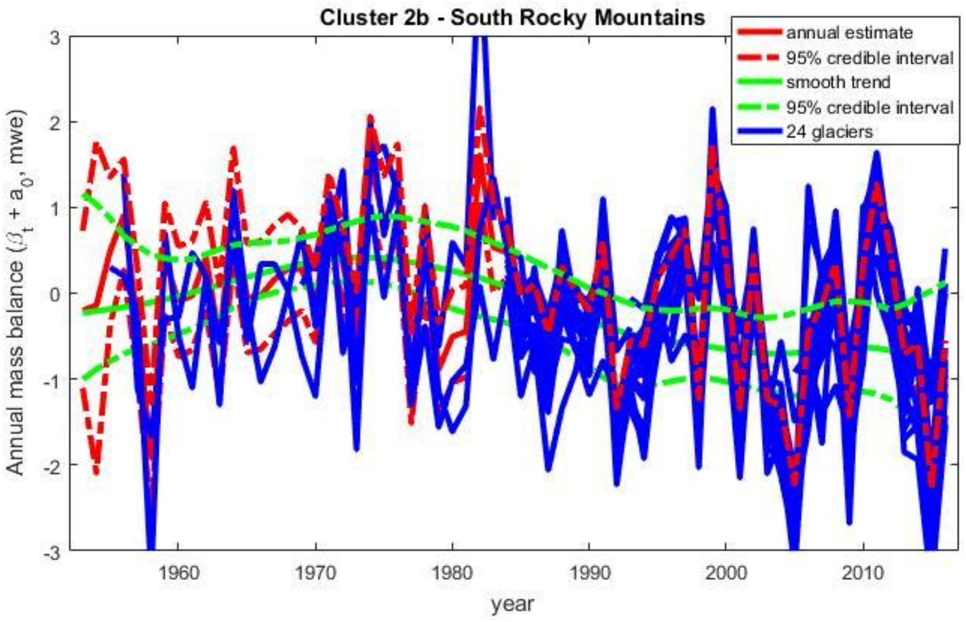
Temporal signal for each group from measurements (2)

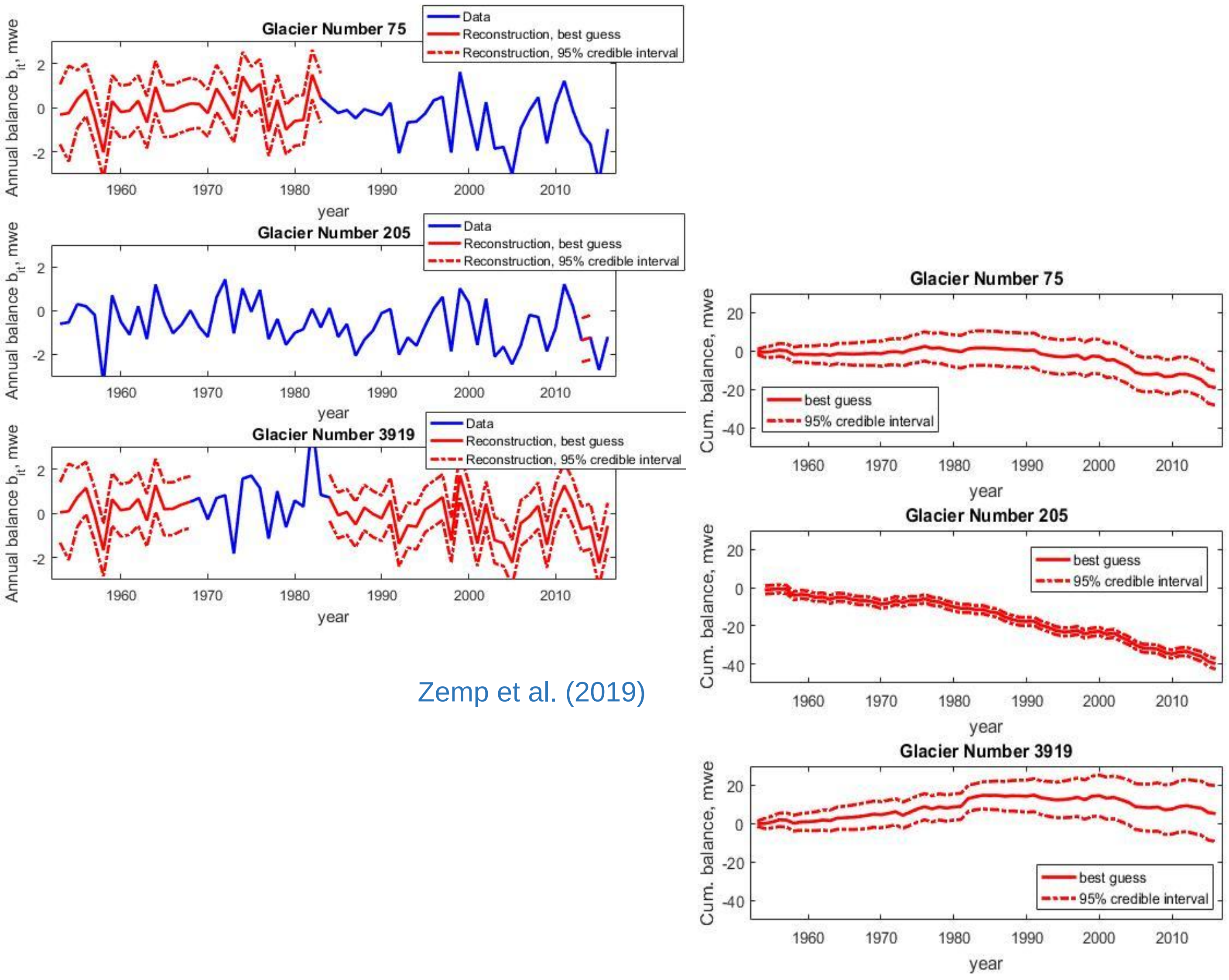
- For each group:

✓ Common temporal structure & underlying trend

✓ "Reconstruction" of full individual series

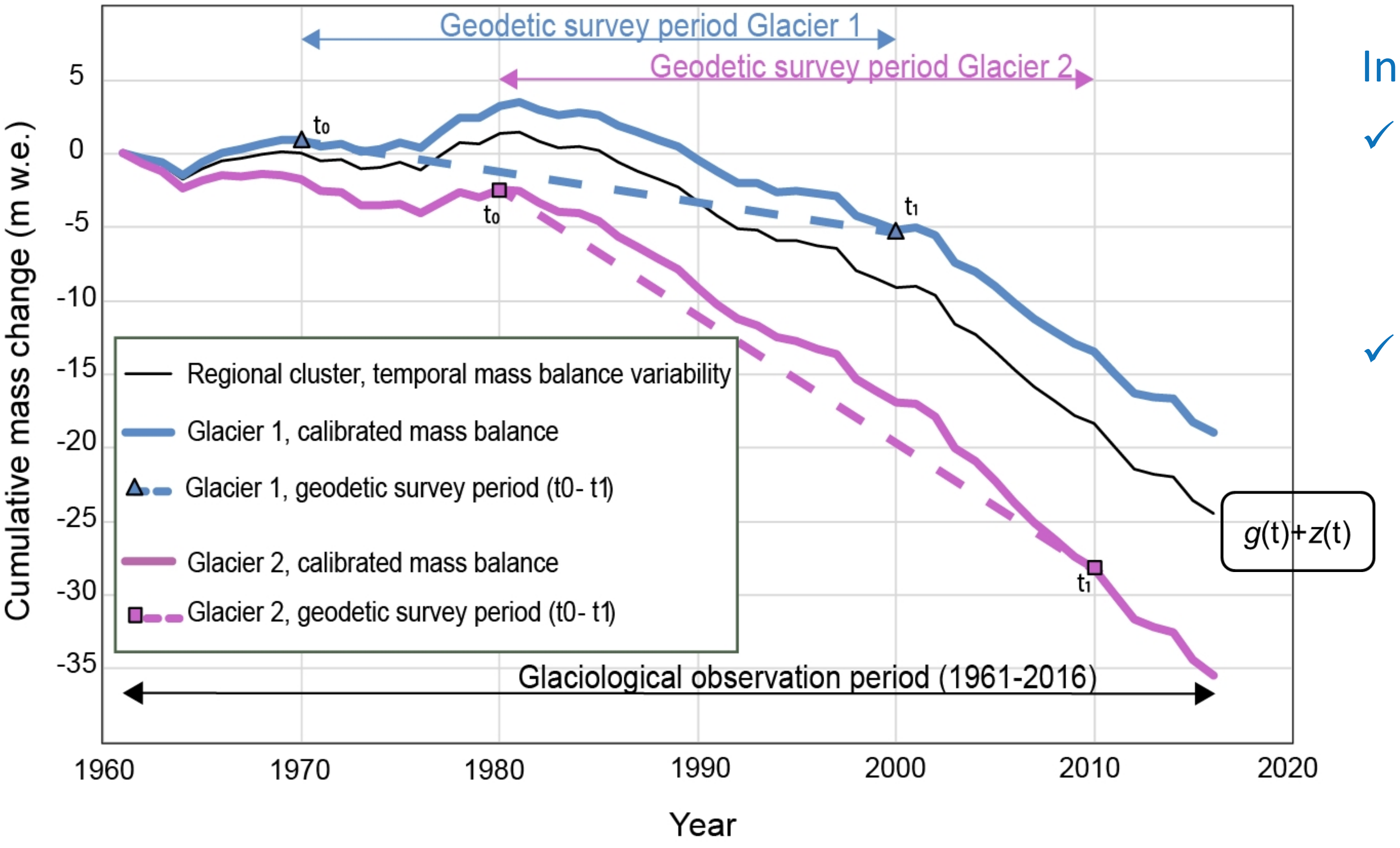
✓ Cumulated changes and associated uncertainty





Zemp et al. (2019)

Combination with remote sensing data

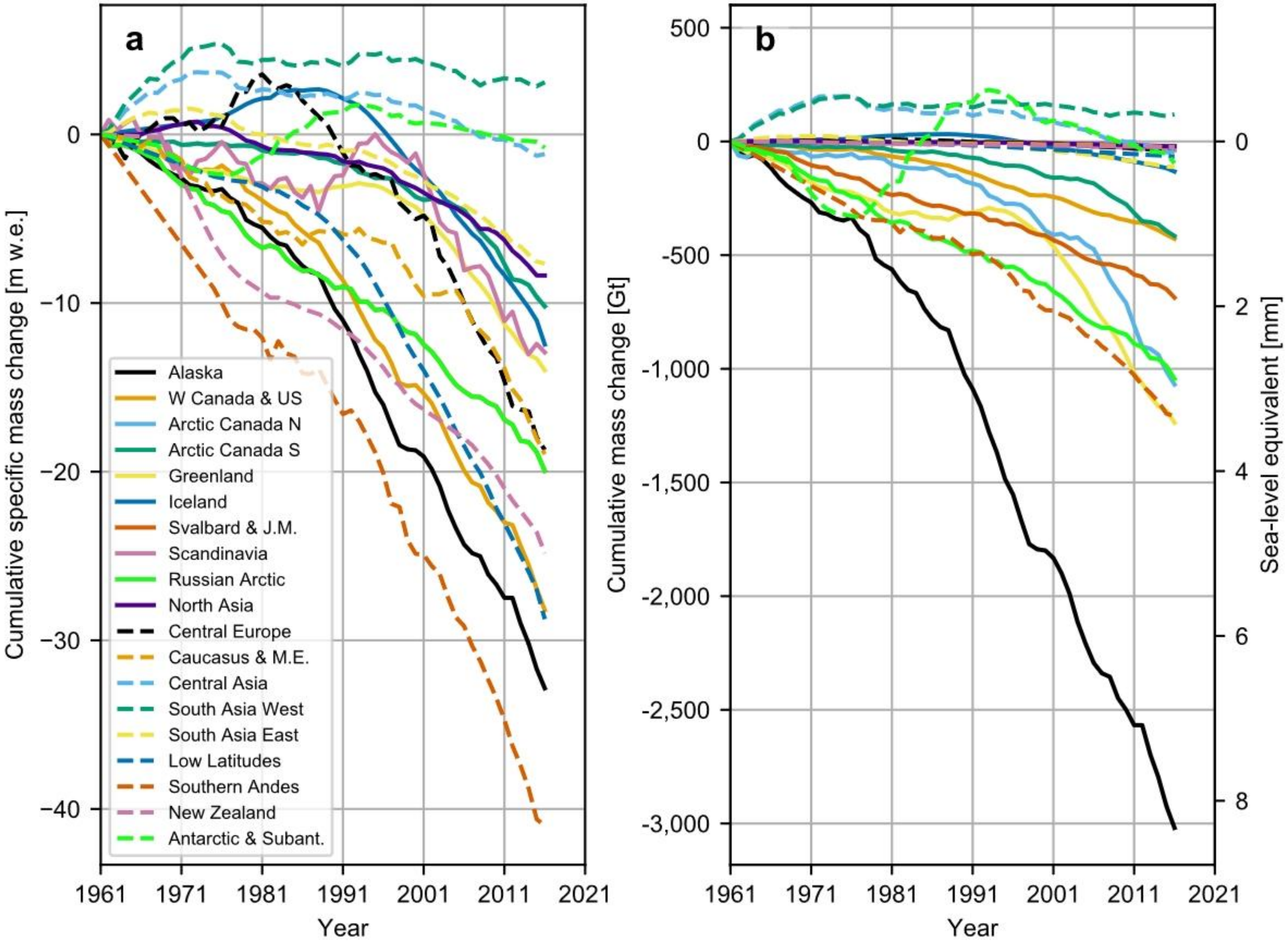


Zemp et al. (2019)

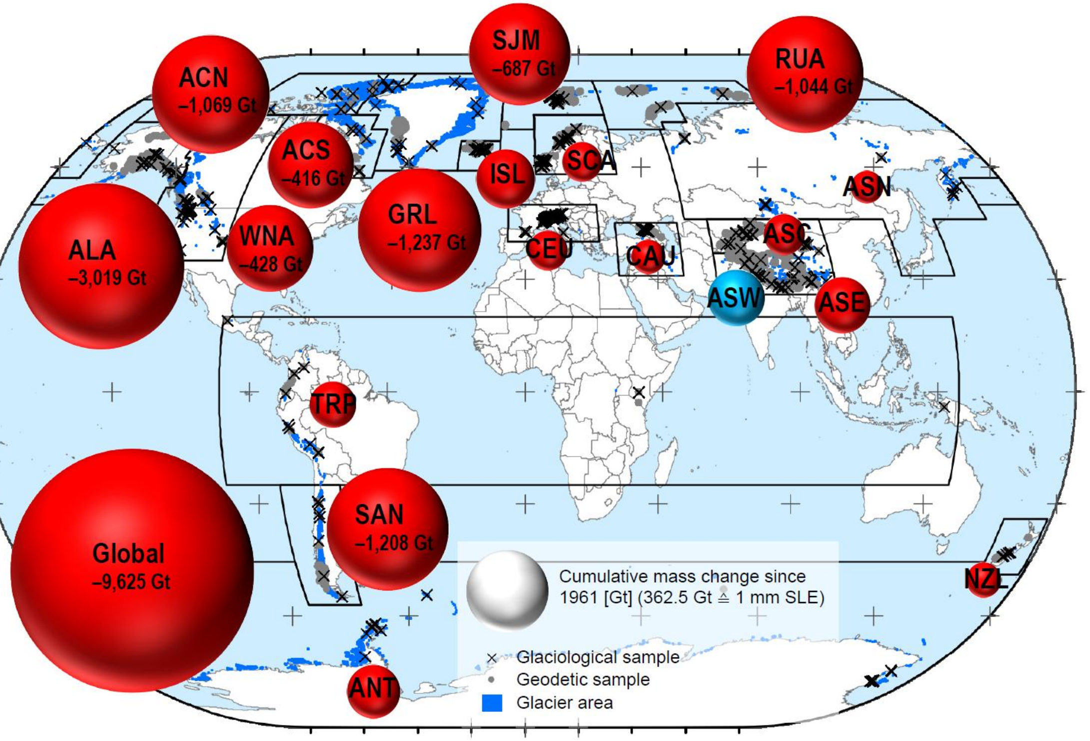
In each group:

- ✓ Combination of temporal structure provided by statistical model with long term changes provided by remote sensing (DEM differencing)
- ✓ Evaluation of ice volume changes taking into account changes in glacier extents

Results (1) : glacier mass change

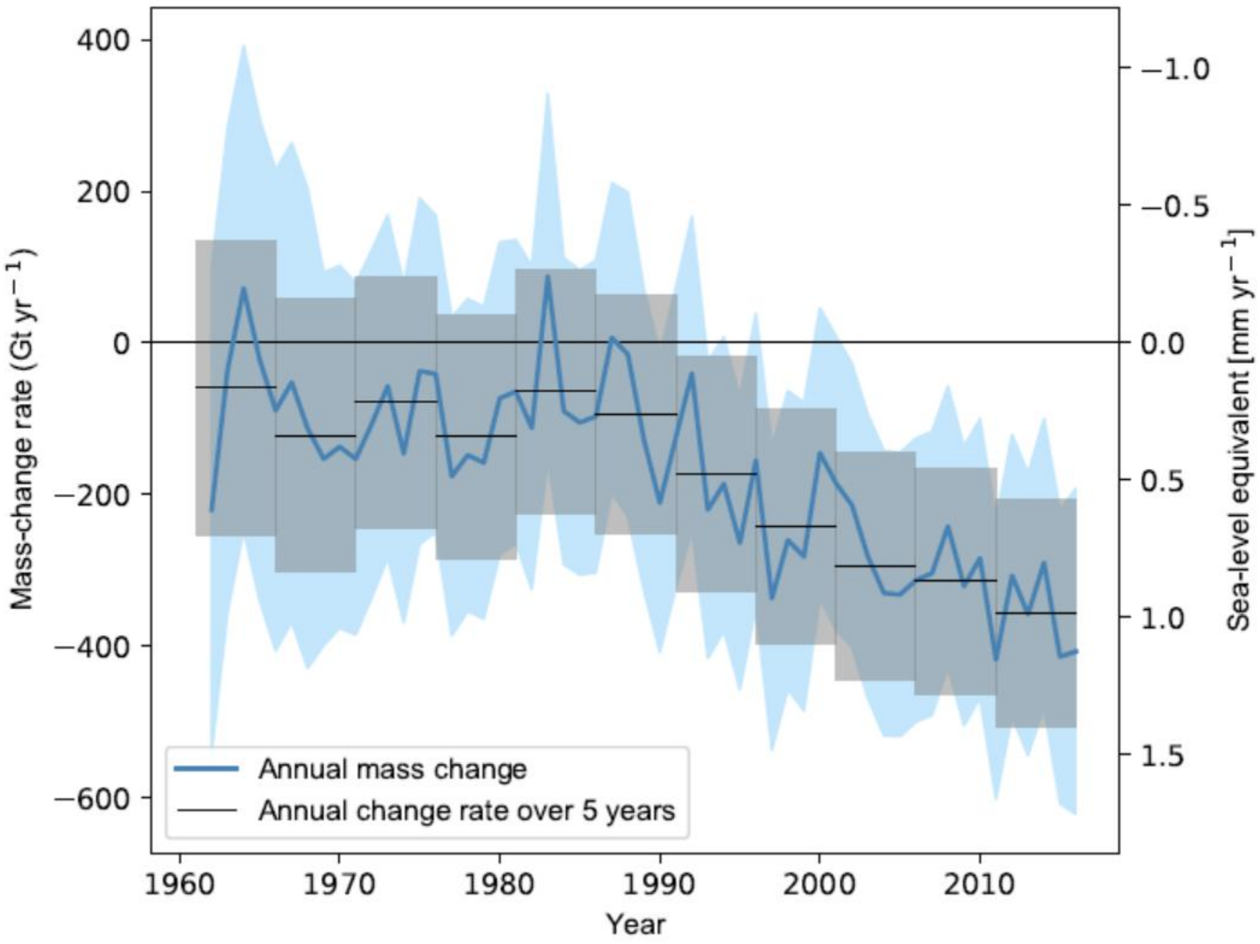


Glacier mass changes rates 1961-2016 (Zemp et al., 2019)

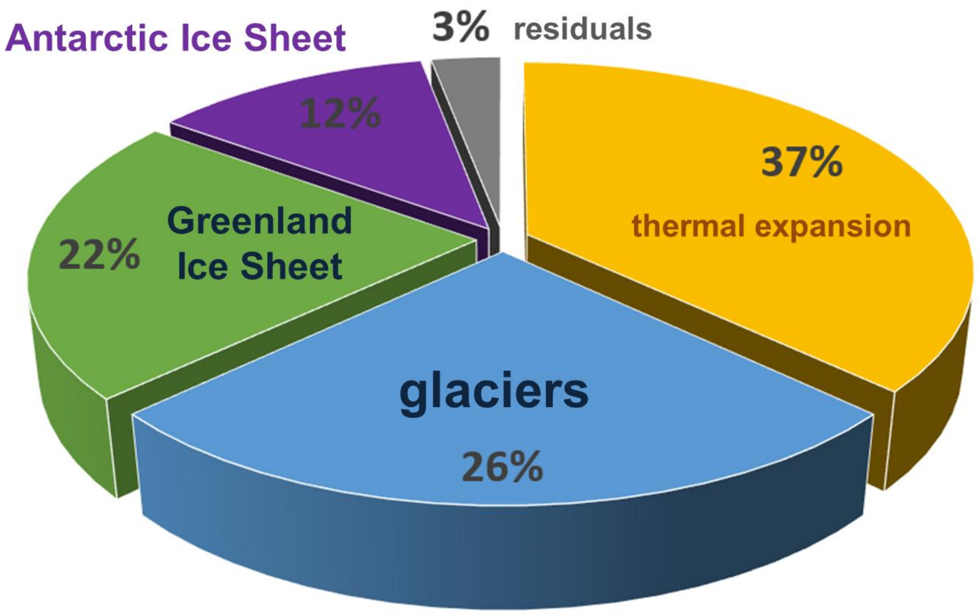


Cumulative glacier mass changes 1961-2016 (Zemp et al., 2019)

Results (2) : contribution to sea level rise

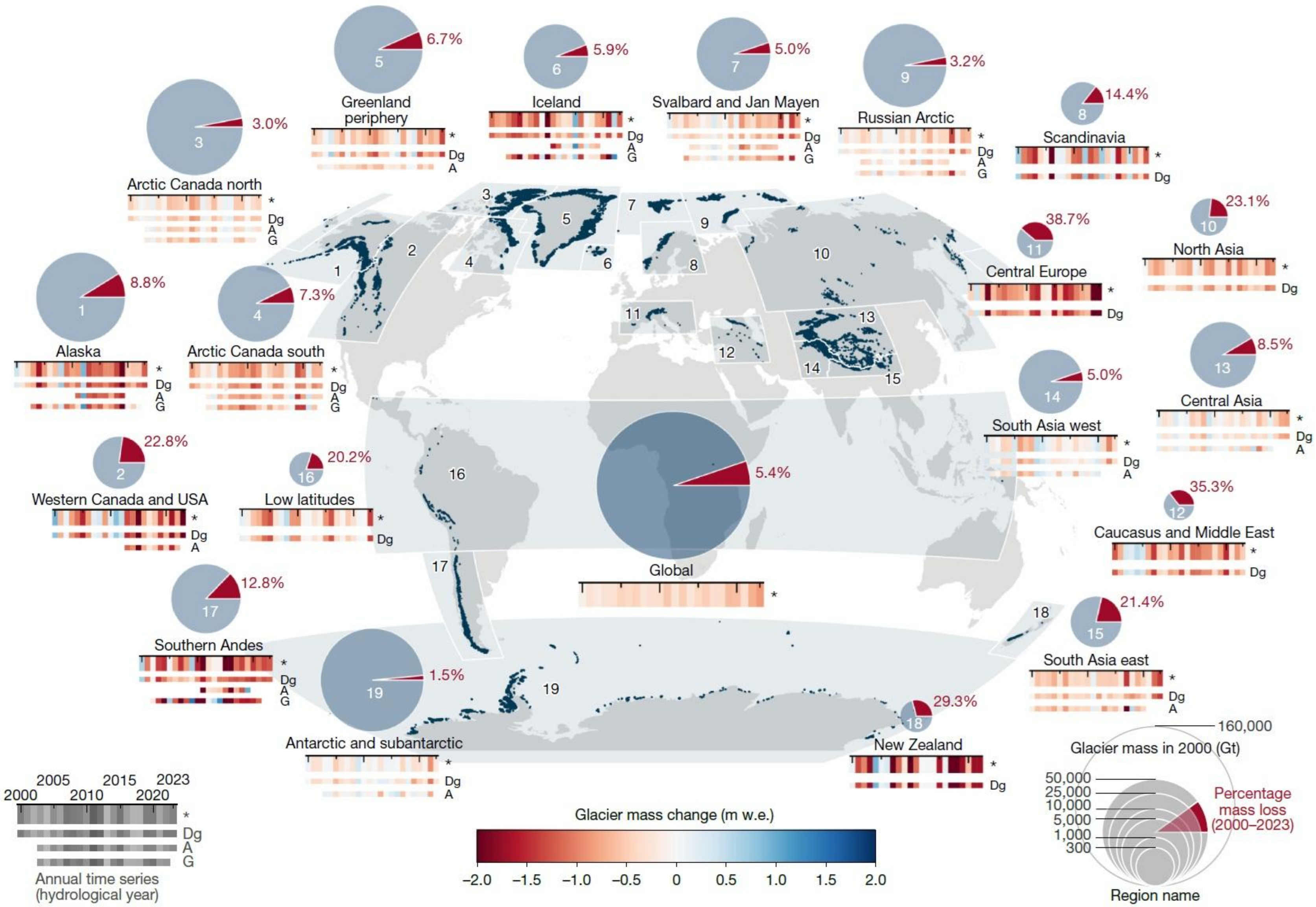


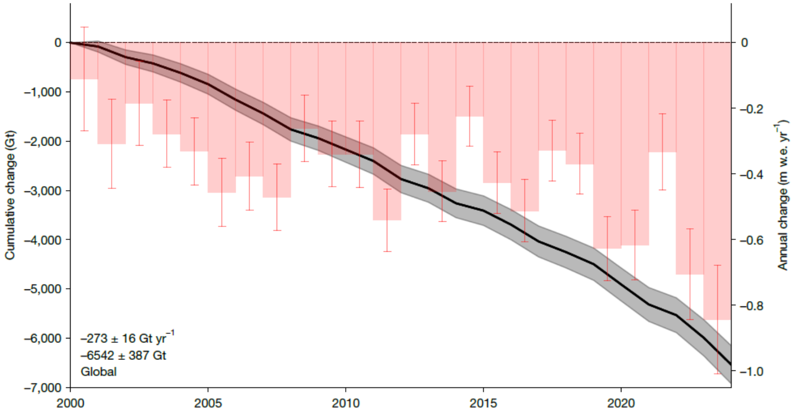
Annual glacier contributions to sea-level rise 1961-2016 (Zemp et al., 2019)



Main contributions to sea-level budget 2004-2015 (Zemp et al., 2019 + data from Cazenave et al. 2018)

A recent update / improvement





More and more recent data with a combination of measurement techniques (Zemp et al., 2025)

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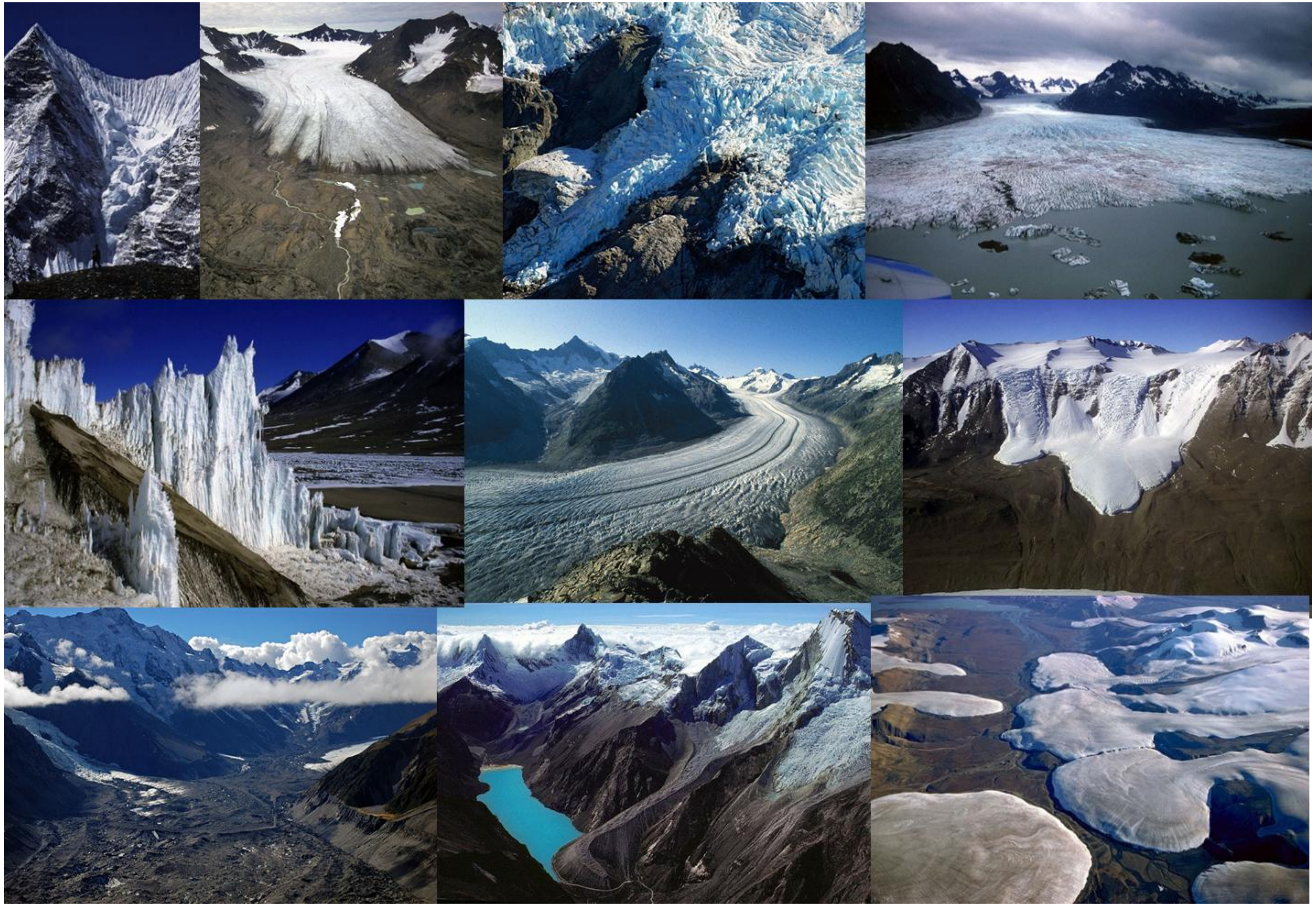
- **Conclusions**



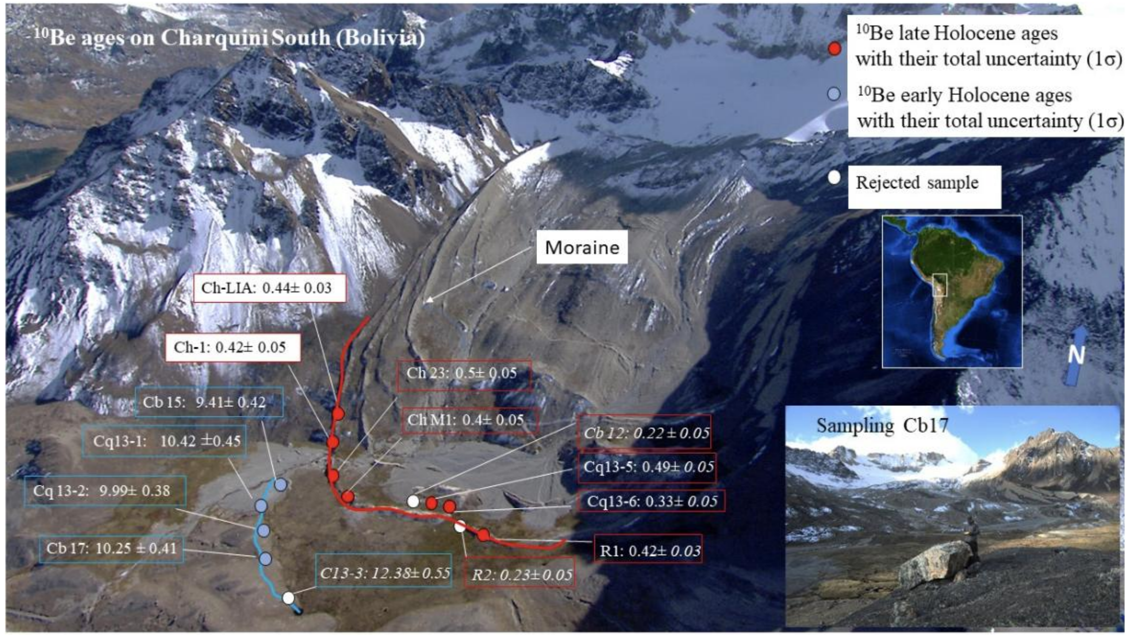
Conclusion (1) : a playground for statisticians

- Important problems:
  - Water resources
  - Sea level rise
  - Disaster risk
  - And more : river temperature (hydropower production, ecosystems)

- Rather comprehensive data sets (WGMS etc.)

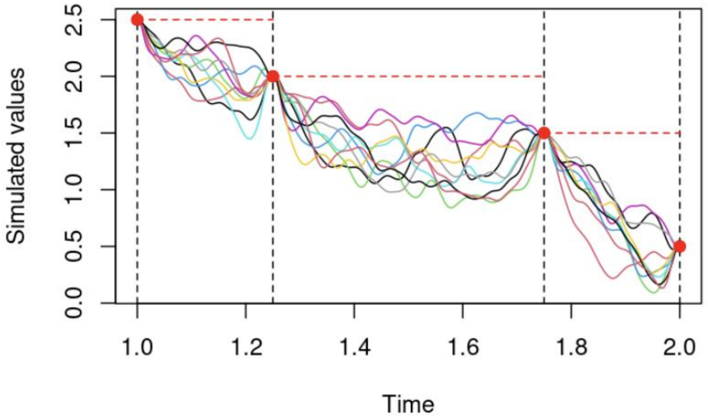
- Very simple models / approaches (my talk)

- Space for developments:
  - Extreme non-stationarity and disaster risk
  - “Complex” data, ex. of Maud Mégret PhD



Reconstitution of long-range glacier fluctuations from moraine position

@ Maud M. Mike P., Philippe N., Vincent J and Nico E.
@ IRIMONT



Conclusion (2) : now or never

- Acknowledgements:
  - for your attention
  - Denis & Thomas (invitation, organisation)

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